The Big Math Ideas

Teaching Mathematics Conceptually K-Algebra 2

Authored by the Keep Indiana Learning Mathematics Team
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The 'Big Math Ideas' document sprouted from years of providing instructional and content support to teachers in Indiana. We all can struggle with the language of the standards, the true meaning of standards, and the connections between standards within a single grade level and between grade levels. Too often we have seen math classrooms where what is being taught is either not part of that grade level's standards at all, is a misinterpretation of the grade level standards, or another grade level's standard being taught in the wrong grade level. Additionally, not all standards should be treated equally. Some standards are low priority standards and require a cursory exploration, while other standards are mastery level standards and require a deep dive into real-world models, connections, and the "why." All of these considerations can result in teacher and student confusion as grade level content is misrepresented. Our standards are written in such a way they provide a progression of mathematical concepts. Each grade level's standards build off the last allowing students to truly develop conceptual understanding of what they are learning. When we don't give them the opportunity to do so, we are keeping them from having authentic success in mathematics and not serving the needs of our students as growing mathematicians.

The 'Big Math Ideas' are the standards seen through a mathematician's approach to the grade level content, rather than a testing approach. All of the "Big Math Ideas" not only meet ILEARN requirements, they show how, when taught properly, they exceed grade level expectations. They also tell a mathematical story at each grade level - a story that includes the knowledge and skills students bring with them to their grade, how those skills are advanced, and how those advancements purposefully prepare them for the grade level to come. Each grade level's 'Big Math Ideas' are represented in 3 different ways: visually, a narrative, and

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indicators of mastery. All three representations share the same 'Big Math Ideas' for the grade level, we just represent the information in different ways allowing everyone to access the information in their own way. Below are some other ideas we ask you to consider throughout this work and are discussed here as an introduction.

Shifts in the Standards

The Indiana Academic Standards shifted in 2020; yet the real change began in 2014 when the standards became College and Career Ready standards. Unfortunately, many educators and textbook publishers have been interpreting these standards as a set of procedures or specific processes to be memorized, rather than a connected and cohesive set of math ideas. The following 'Big Math Ideas' bring together the Indiana Academic Standards, the college and career ready standards, and the mathematics teaching and learning shifts called upon by NCTM to define what students really need to understand about math. They are the how, the what, AND the why of K-12 mathematics.

Teaching Mathematics Conceptually

Teaching mathematics conceptually means we build a math classroom where the following are true:

- Students' voices are encouraged, celebrated and meaningfully elicited and incorporated into the lesson.
- Variation in mathematical strategies is always a part of the learning process; student initiated, teacher initiated, or curricular initiated strategies are all celebrated equally.
- Modeling with manipulatives and diagrams, drawing, technology, and anything that helps students make sense of mathematics is the first step in learning.
- Students' understanding drives the learning process, hence frequent formative assessments are vital.

This list could continue on; yet the purpose of this vast document and each of its components is to support Indiana's K-12 math educators and leaders in their pursuit of teaching mathematics conceptually within the framework of the current Indiana Academic Standards (IAS) and standardized testing system. This document is

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informed by the IAS and the ILEARN Priority Standards; however, it goes much further by thinking beyond what students need to do and digging deeper into what students need to understand within each grade level. So often we, as educators, can get caught up in 'I Can' statements and what skill or procedure a student can perform. The 'Big Math Ideas' of each grade level identify for educators what students need to mathematically understand, not just be able to do." There is a distinct difference between the two, and this document strives to support educators in identifying the key understandings their students should develop within and across grade levels.

Defining Mastery

Within this document, you will find the use of the word "mastery, and we believe this term must be defined. When we identify a concept as a mastery standard or mastery concept for a grade level, we mean a student should have a surface, deep and transfer level of understanding of that concept.

Surface Learning: Foundational understanding. Students can define the concept in their own words or identify the concept if shown examples.
Deep Learning: Students can apply their knowledge to a real-world situation or share their understanding with a peer.
Transfer Learning: Students have a conceptual understanding of the concept allowing them to apply their learning to a new and unique situation they have not experienced before.

If a standard is identified as a "low" level standard, the student would be expected to have a surface level of understanding. If a standard is identified as "medium" level, a student would have surface and deep level learning of that standard. "High" level or mastery level understanding is all three levels of learning.

Within this document, we have identified the instructional significance of each standard (High, Medium, and Low) and the indicators of mastery for that standard. Both of these determinations were made through the analysis of where that standard or skill is mastered in the K-12 progression of skills. Please note this work was informed by the ILEARN assessment blueprints (what will most likely be assessed), but our identification of instructional significance was also **informed by what students will need to know and understand about math, even if the skill itself**

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is not highly assessed. The identification of “instructional significance” will help practitioners to determine how important a specific skill or standard is for their grade level. Our goal with this indicator is to help educators to determine which skills to dig deeply into with their students. The “Indicators of Mastery” will define what students will understand about the concept and provide a clearer definition of the ways students will show their understanding.

Modeling

Modeling mathematics for students and helping them to learn to model mathematics is critical to conceptual understanding of mathematics. Sixty-five percent of students are visual learners. Students are immersed in visual information from television, the internet, and their phones. Their brains are almost programmed to take in visual data and interpret it quickly. We can ignore that or use it to our advantage. When we pair information, definitions, procedures, etc., with visuals, we make it easier for the concept to get into long term memory(shiftlearning.com).

Visuals, including models we use in mathematics, affect learners on a cognitive level. "When words and visual elements are closely entwined, we create something new and we augment our communal intelligence ... visual language has the potential for increasing ‘human bandwidth'—the capacity to take in, comprehend, and more efficiently synthesize large amounts of new information" (Robert E. Horn, Stanford University).

There is also a biological reason to teach mathematics through models. Visuals are encoded in the brain in the same place where emotions are processed. This creates a stronger memory and the idea or concept being taught is retained longer (shiftlearning.com).

Modeling mathematics is also about equity and access. Many students from procedural classrooms learn at a very early age they are not “mathy” or they are “not a math person” because teaching procedurally is like trying to squeeze everyone in the front door, and only those with the key (knowing the one procedure or formula taught) can get in. Modeling gives students access to mathematics in other ways. It’s like handing them a key to the side entrance, or letting them know that it’s ok to climb in the back window. Modeling gives more students a chance to see themselves as mathematicians and authors of their learning.

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There are a myriad of examples of what modeling in mathematics means. Throughout all of our documents, you'll be provided with examples of modeling. At the elementary level, you'll see examples of manipulatives that are used to model counting, numbers and computation. Items with which you can count, represent numbers, and compute are literally endless. One can count anything and label it with its numeral; however, typical items we might see in classrooms are unifix cubes, one-inch tiles, two-sided chips, counting bears, pattern blocks, etc. As students begin representing numbers, all of the aforementioned items can be used, but students will also be introduced to base ten blocks. Units, rods, flats, and cubes can be used to represent ones, tens, hundreds and thousands. Later, these same tools can model decimals. When the flat becomes the value of one, the rod becomes a tenth and the unit a hundredth. Fractions can be modeled with pattern blocks, unifix cubes, counting bears - the possibilities are endless - and they can all be used to model all four operations in computation.

Other models include the use of number lines to represent numbers, compare numbers, and compute numbers. Arrow language can model one's mathematical thinking as they use various operations to solve problems. Ratio tables can represent multiplication and division.

The key to understanding modeling is that all of the various representations children use to share their mathematical thinking are how they are making sense of the math they are doing. Modeling, and honoring children's mathematical thinking through various representations, is crucial to their development of the conceptual understanding of the many math concepts they encounter in school.

Importance of reading outside of your grade level (above and below)

The strength of this document is in the way it creates a comprehensive view of K-12 mathematics for educators and leaders. By looking across grade levels and considering where big ideas are introduced, reinforced, and mastered, we can more clearly understand how to support our students through their learning experience. It is of vital importance educators and leaders read within and outside of their grade-level throughout this document. At a minimum, we encourage practitioners to read materials for their own grade level and to read the materials for one grade level above and one grade level below. In order to understand what we want our students

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to know and understand at our own grade level, we must also understand what students learned before they entered our grade level AND what they will learn when they move to future grades. Widening our view of the teaching of mathematics will help to ensure that the 'Big Math Ideas' and big understandings of mathematics content are aligned across all grade levels. This comprehensive, cohesive approach to teaching mathematics is key to success for our students.

Included documents

We hope that the work we have done and the documents we are providing help to build content knowledge and instructional clarity for educators in Indiana. Our resources include:

- **Grade Level Visual Big Math Ideas** provides a visual representation of how the concepts are weighted and prioritized but also how they are connected and interwoven together.
- **Grade Level Narratives** providing an overview of the Big Math Ideas within each grade level. The Big Math Ideas represent the important mathematical understandings that should be focused on and solidified during that grade level.
- **Standards Instructional Significance and Mastery Indicators** documents for each grade level. These documents identify the importance of each standard, based on its alignment to the important mathematical ideas. These documents also include Mastery Indicators that describe what the standard means and what students should know and understand within each standard.
- **Progressions Documents** for K-2 Number Sense; K-4 Addition; K-4 Subtraction; K-5 Multiplication; K-6 Division; K-6 Fractions, Decimals, and Percents as Numbers; 4-6 Computing with Fractions; 6-8 Equations and Expressions; and 6-8 Ratios and Proportions. These documents provide an in-depth exploration of the skills and concepts students are learning across grade levels. These progressions are intended to support educators in understanding the knowledge students are coming to their classrooms with and to provide insight into how students will grow in that knowledge in the next grade levels. (Coming soon!)

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Kindergarten-Fifth Grade Big Math Ideas

The Keep Indiana Learning Team unpacked the standards and Big Ideas of K-5 mathematics and created resources to help teachers increase their content knowledge in order to best support students. The purpose of these resources is to transform teaching and learning of mathematics by increasing educator math content knowledge, identify the most important ideas within each grade level to provide instructional focus for teachers and schools, and clearly define the exact knowledge and understandings students should have for each standard. Throughout our work, we identified a few key understandings about the K-5 content:

1. Across our resources in the K-5 grades, there is a consistent emphasis on number sense and computation. This work includes specific content development across ALL operations in ALL grade levels. Beginning in the primary grades, students will interact with addition, subtraction, multiplication, and division of whole numbers. When moving into the intermediate grades, students will also start to develop number sense and computation with fractions and decimals. Educators should pay careful attention to the specific computation strategies, models, and number types students are expected to master within each grade level, with a specific focus on knowing when students should be using strategies and when they should be ready to be introduced to algorithms. It is important teachers align their instruction to the standards in order to create a cohesive and cumulative learning experience which meets NCTM's recommendation of building procedural fluency from conceptual understanding.

2. In addition to considering computation strategies and algorithms, students in grades K-5 should have learning experiences which are grounded in real-world problems. This means students are interacting with realistic and relatable problems. The purpose of grounding learning in real-world problems is to help students develop an understanding of the computational thinking in the problems they are solving. If problems are unrealistic or unrelatable, students will not be able to connect to the problem in ways which create meaningful experiences for learning. The numbers within the real-world problems in any given grade-level will also change across the school year, as students learn more and are able to interact with more complex numbers and problems. An additional consideration

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with real-world problems is to ensure students are provided with experiences with all [14 problem types](). Students should NOT engage with only one problem type at a time; instead, creating a balance of mixed problem types across the year, with numbers becoming more complex as the year progresses, is an important aspect of elementary math experiences.

3. Finally, K-5 student experiences will build understandings of geometry, time, money, measurement, and data collection and analysis. This work is often integrated through real-world problem solving work and number sense activities. Additionally, there are specific skills students should learn within each grade-level, which may take place outside of problem-solving experiences. These skills should be taught and reinforced as needed, in order for students to develop key content understandings.

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Math Concept Progressions

The Keep Indiana Learning Big Math Ideas Math Concept Progressions are the map guiding educators through all of the connections and interwoven networks built into the mathematical content within the Indiana State Standards. These progressions support educators in understanding how math concepts are taught, learned, and developed across grade levels.

They provide:

- a **navigational compass** to direct educators from grade level to grade level within a key mathematical concept
- **key landmarks** along the pathway as students develop an understanding of operations, fractions, number sense, and other important mathematical underpinnings
- an **overarching picture** of the beginning, middle and end of the mathematical thinking in which a student develops

Click on the Math Concept Progressions below to get started:

- Kindergarten - 2nd Grade: *Number Sense*
- Kindergarten - 4th Grade: *Addition*
- Kindergarten - 4th Grade: *Subtraction*
- Kindergarten - 6th Grade: *Multiplication*
- Kindergarten - 6th Grade: *Division*
- Kindergarten - 6th Grade: *Fractions, Decimals, and Percents as Numbers*
- 4th - 6th Grade: *Fractions and Decimals Computation*
- 6th - 8th Grade: *Expressions/Equations*
- 6th - 8th Grade: *Ratios and Proportions*

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Big Math Ideas

Kindergarten

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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with

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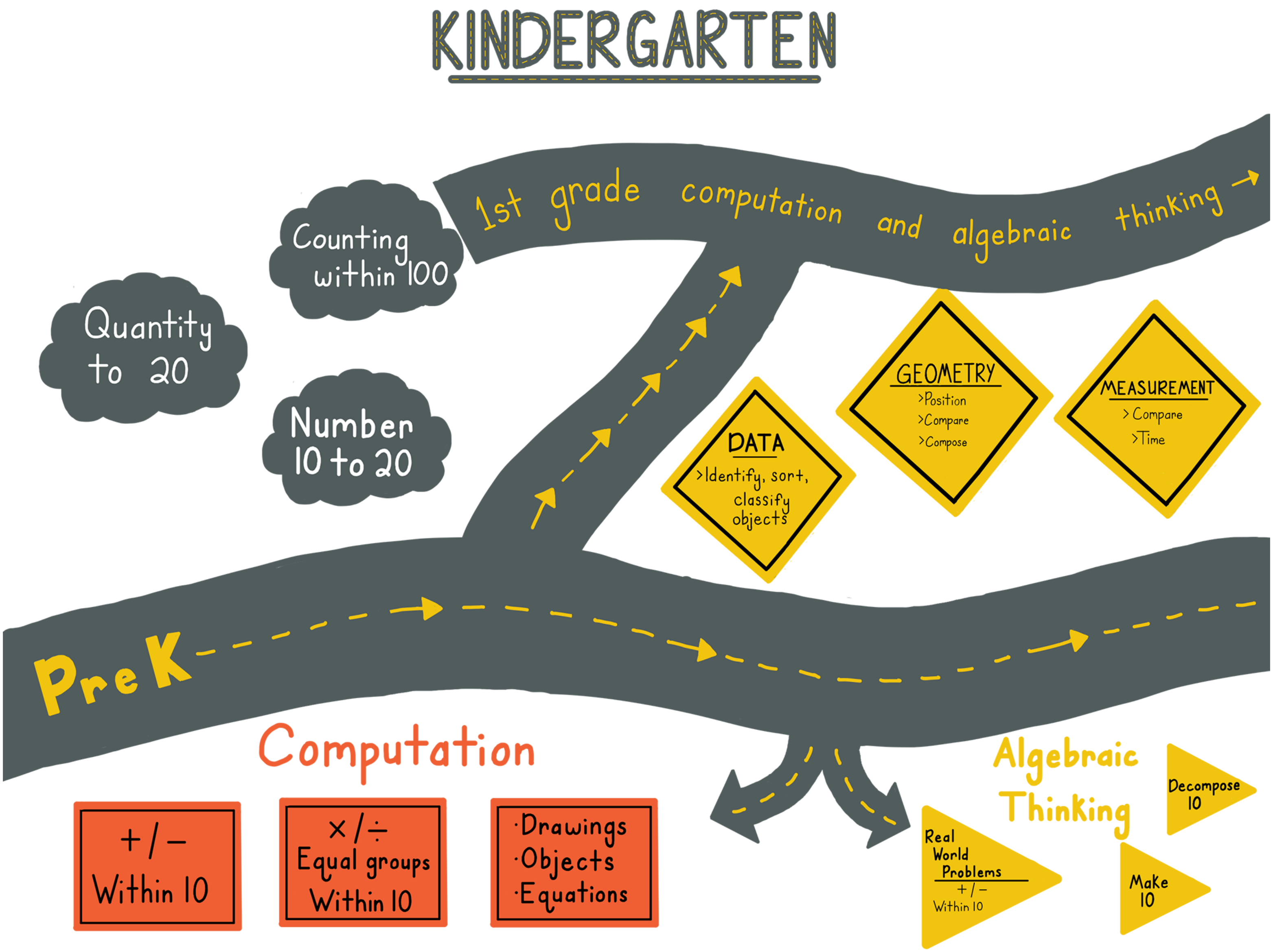
Kindergarten Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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Kindergarten Big Math Ideas - Narrative

Number Sense

The biggest work happening in the Kindergarten year is helping students develop a strong number sense foundation. This work will set them up for success in future grades. Students should have lots of experience building, representing, counting, arranging, describing numbers. Give students time with activities that encourage them to organize and count objects. It is important to support them in truly understanding the important work of counting and that a numeral represents a specific quantity. It can be tempting to move quickly through number sense skills in Kindergarten and to focus on memorizing the counting sequence, but this leads to a very surface-level understanding. Instead, make sure counting has a constant presence in the Kindergarten classroom across the entire year. Students should not only leave Kindergarten with memorization of the counting sequence, but they should understand that a numeral has a specific quantity. Be sure to visit our K-2 Number Sense Progression (coming soon) to see how your students will build on this work in future grades as they extend the counting sequence and use skip-counting in their computation strategies.

Computation and Algebraic Thinking

Kindergarten students should also spend significant time building their understanding of computation and algebraic thinking. They should engage in experiences with all four operations, with a larger focus on addition and subtraction. Keep in mind Kindergarten students can and should develop addition and subtraction strategies. All year long, students should be solving real-world addition and subtraction problems using objects and drawings. Students should use concrete manipulatives and tools to model the real-world problem as they develop an understanding of adding (combining sets) and subtracting (finding the difference between two sets or separating values). They should be supported in understanding and using ten-frames, number lines, counters, and other tools to help model the composing and decomposing work they are doing in their combining and

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separating. Specific attention should be paid to combinations of ten, as the ability to make ten from a given number is an important concept students will use over and over again, and they will extend this understanding to combinations to multiples of tens and hundreds. They should solve problems with unknowns in ALL parts of the problem (start, change, result):

- Unknown Start: I had some crayons. I gave 4 to my friend. Now I have 6. How many crayons did I have before I gave some to my friend?
- Unknown Change: I had 10 crayons. I gave some to my friend. Now I have 6 crayons. How many crayons did I give my friend?
- Unknown Result: I had 10 crayons. I gave 4 to my friend. How many crayons do I have now?

Solving problems with the unknown in all parts of the problem will help students to understand that we aren't always solving for the "end" to a problem. The teacher should show students the equations that match these problems, but students should not be required to write the equations until they show a strong understanding of the computation work. When showing and teaching about equations, be careful not to teach into the common misconception that the equal sign means that the answer will follow. This is a "rule" which expires very quickly as students begin to learn to identify true and false expressions in first grade. To learn more about how the computation work in Kindergarten is connected to work in future grades, be sure to visit our Addition and Subtraction Progressions (coming soon).

Kindergarteners will also be introduced to multiplication and division through the organizing of objects into a specific number of groups or into groups with a specific value. Use physical objects for this work! Students need time to explore and understand how to create equal groups. This will include a lot of time and practice, and they deserve to be given this time. Our Multiplication and Division Progressions (coming soon) includes detailed information about how the multiplication and division work Kindergarten students do is connected to later grades.

Number Sets

It is important to carefully consider the number size you are using when supporting specific skills with your students. Some number sets will be used to

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support Number Sense skills, while other sets are used in computation. Here is a summary of the work Kindergarten students will do with numbers of specific sizes:

Number Size	Skills
Numbers to at least 100	count by 1s and 10s from any given number
Numbers within 20	write whole numbers find one more/one less count objects arranged in line/array/circle compare two sets, compare two numerals use a numeral for the number in a set show equivalent forms of whole numbers
Numbers within 10	number words count scattered objects recognize the value of a patterned set without counting create equal groups addition and subtraction, including real-world problems decompose in more than one way find combinations to 10

Additional Learning

Kindergarten students will also spend time with geometry, data, and measurement skills. This work should focus on exploration and helping to set students up to discover ideas about geometry, data, and measurement. Let students explore shapes and consider the number of sides they have. Help them to start to understand how shapes can be composed and decomposed by encouraging them to put together shapes into larger shapes and to break apart larger shapes into smaller shapes. This work is foundational for fraction knowledge, as described in our K-5 Fraction Progression (coming soon).

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Students should also have exploration time as they compare objects by length and weight, using comparison words when describing them. Allow students to find ways to sort and classify objects. All of this work will support students in developing initial understandings of these concepts. However, this work should not take up the majority of the school year. Consider using other parts of the school day to integrate some of these skills across the year.

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Kindergarten Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
	K.NS.1 Count to at least 100 by ones and tens and count on by one from any number.	High	Students will understand that numbers follow a sequence and will apply that sequence starting from any number. Students will be able to count by ones to 100. Students will be able to count by tens to 100.
	K.NS.2 Write whole numbers from zero to 20 and recognize number words from zero to 10. Represent a number of objects with a written numeral zero to 20 (with zero representing a count of no objects).	High	Students will understand that whole numbers are written with numerals. Students will understand that a numeral represents a value. Students will write whole numbers to 20 and recognize word form up to 10. Students will write a numeral to match a set up objects up to 20.
	K.NS.3 Find the number that is one more than or one less than any whole number up to 20.	High	Students will understand the counting sequence. Students will apply the understanding of sequence by telling a number that come just before or just after a number with 20.
	K.NS.4 Say the number names in standard order when counting objects, pairing each object with one and only one number name and each number name with one and only one object. Understand that the last number describes the number of objects counted and that the number of objects is the same regardless of their arrangement or the order in which they were counted.	High	Students will understand that counting objects follows a number sequence. Students will understand that when counting they should say one number name for each object. Students will understand that the last number they say when counting represents the items in the collection. Students will count a set of objects up to 20 (K. NS.3) using one number name for each object. Students will use the last number said to tell how many objects are in a collection.
	K.NS.5 Count up to 20 objects arranged in a line, a rectangular array, or a circle. Count up to 10 objects in a scattered configuration. Count out the number of objects, given a number from one to 20.	High	Students will develop strategies for counting collections of objects. Strategies for organizing can include but are not limited to, arranging in a straight line, making and array, or organizing them in a circle. Students will understand how to count out objects to represent a number. Students will be able to count up to 20 objects that are arranged in a non-scattered configuration. Students will be able to count up to 10 objects in a scattered configuration.

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Number Sense	**K.NS.6** Recognize sets of one to 10 objects in patterned arrangements and tell how many without counting.	High	Students will understand that they don't always have to count a collection to know how many objects there are. Within 10 and in a patterned arrangment, students will be able to identify how many are in a collection.
	K.NS.7 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g. by using matching and counting strategies).	High	Students will use strategies (such as lining up, counting, etc.) to compare two groups. Within 20 (K.NS.5), students will be able to use startegies to compare two groups.
	K.NS.8 Compare the values of two numbers from 1 to 20 presented as written numerals.	High	Students will understand that a numeral represents a value. Students will understand the sequence of counting. Students will understand that, when counting, the number said represents the value of that number. Students will tell if written numerals are greater than/less than/equal to and explain how they know.
	K.NS.9 Correctly use the words for comparison, including: one and many; none, some and all; more and less; most and least; and equal to, more than and less than.	Medium	Students will understand that mathematicians use specific words to compare a number or number of obejects. Students will know the comparison words: one, many, none, some, all, more, less, most, least, equal to, more than, less than. Students will use comparison words when comparing sets of objects and numerals.
	K.NS.10 Separate sets of 10 or fewer objects into equal groups.	High	Students will understand "equal" and "unequal" groups. Students will understand that objects (up to 10) can be separated into more then one group. Students will be able to separate objects (up to 10) into two groups and tell if the groups are equal or unequal. Students will equally separate objects into equal groups.

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	K.NS.11 Develop initial understandings of place value and the base 10 number system by showing equivalent forms of whole numbers from 10 to 20 as groups of tens and ones using objects and drawings.	High	Students will understand that numerals (up to 20) represent a number of tens and ones. Students will understand that numerals must be written so that the number in each place is the correct value. Students will understand the numbers can be represented (with objects and drawings, including but not limited to Unifix cubes, counters, base-ten blocks) in more than one way using tens and ones.
Computation and Algebraic Thinking	**K.CA.1** Use objects, drawings, mental images, sounds, etc., to represent addition and subtraction within 10.	High	Students will understand that addition is the combining of two or more values. Students will understand that subtraction is finding the difference between values. Students will understand that they can use startegies such as objects (such as counters, Unifix cubs, and base-ten blocks), drawings, mental images, and sounds to combine or find the difference between two numbers. Within 10, students will represent addition and subtraction using drawings, mental images, sounds, etc.
	K.CA.2 Solve real-world problems that involve addition and subtraction within 10 (e.g., by using objects or drawings to represent the problem).	High	Students will understand that addiiton is the combining of two or more values. Students will understand that subtraction is finding the difference between values. Students will use objects, drawings, etc. to represent the combining or separating of numbers. Students will solve real-world problems using addition and subtraction.
	K.CA.3 Use objects, drawings, etc., to decompose numbers less than or equal to 10 into pairs in more than one way, and record each decomposition with a drawing or an equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). [In Kindergarten, students should see equations and be encouraged to trace them, however, writing equations is not required.]	High	Students will understand that numbers can be decomposed (broken apart) into smaller numbers. Students will understand "equal" and "unequal". Students will understand that an equation can be used to represent "equal" and "unequal". Students will demonstrate mastery by decomposing numbers in momre than one way. Students will be able to record a decomposition using drawings. Students are not yet required to write equations, but should be exposed and encouraged to trace them.

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	K.CA.4 Find the number that makes 10 when added to the given number for any number from one to nine (e.g., by using objects or drawings), and record the answer with a drawing or an equation.	High	Students will understand that there is more than one way to represent a number. Students will understand how to find a number that makes 10 when added to another number by using strategies such as drawings or pictures. Students will demonstrate mastery by finding the number that makes 10 when given another number. Students will also record the answer using drawings and, eventually, equations.
	K.CA.5 Create, extend, and give an appropriate rule for simple repeating and growing patterns with numbers and shapes.	Medium	Students will understand that patterns are built from repeated changes. Students will understand that patterns can be created or found. Students will know that patterns can be made of repeated changes in shapes or numbers. Students will create patterns using repeition of shapes or numbers. Students will extend a given pattern and explain why they extended it as they did. Students will give rules for patterns and explain why they believe it is the rule.
Geometry	**K.G.1** Describe the positions of objects and geometric shapes in space using the terms inside, outside, between, above, below, near, far, under, over, up, down, behind, in front of, next to, to the left of and to the right of.	Medium	Students will know the meaning of: inside, outside, between, above, below, near, far, under, over, up, down, behind, in front of, next to, to the left of, to the right of Students will use the words to tell how descibe where objects and shapes are.
	K.G.2 Compare two- and three-dimensional shapes in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	Medium	Students will differentiate between two-dimentional and three-dimentional shapes. Students will use informal language to talk about two-dimentional and three-dimentional shapes, such as describing their sides, corners, and faces. Students will be able to identify attributes such as size, vertices/corners, equal lengths, etc. Students will compare shapes using informal language (sides, corners, edges, vertices, faces) about attributes.

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	K.G.3 Model shapes in the world by composing shapes from objects (e.g., sticks and clay balls) and drawing shapes.	Medium	Students will understand that shapes can be composed. Students will understand that shapes in the real-world can be composed and drawn. Students will be able to create real-world objects using shapes. Students will draw real-world objects using shapes.
	K.G.4 Compose simple geometric shapes to form larger shapes (e.g., create a rectangle composed of two triangles).	High	Students will understand that shapes can be composed of other shapes. Students will apply their understanding to compose larger shapes from smaller shapes.
Measurement	**K.M.1** Make direct comparisons of the length, capacity, weight, and temperature of objects, and recognize which object is shorter, longer, taller, lighter, heavier, warmer, cooler, or holds more.	Medium	Students will know the words: shorter, longer, taller, lighter, heavier, warmer, cooler, holds more. Students will understand that items can be compared based on length, capacity, weight, and temperature. Students will demonstrate mastery by comparing objects based on the above characteristics.
	K.M.2 Understand concepts of time, including: morning, afternoon, evening, today, yesterday, tomorrow, day, week, month, and year. Understand that clocks and calendars are tools that measure time.	Medium	Students will develop an understanding that we use time to organize days, weeks, months, and years. Students will understand morning, afternoon, evening, today, yesterday, tomorrow, day, week, month, and year. Students will recognize that clocks and calendars are tools that can be used to measure time.
Data Analysis	**K.DA.1** Identify, sort, and classify objects by size, number, and other attributes. Identify objects that do not belong to a particular group and explain the reasoning used.	Medium	Students will understand what it means to identify, sort, and classify. Students will identify objects that belong and do not belong to various groups and explain their reasoning. Students will use their undertanding of attributes (K.G.2) to identify, classify, and sort objects.

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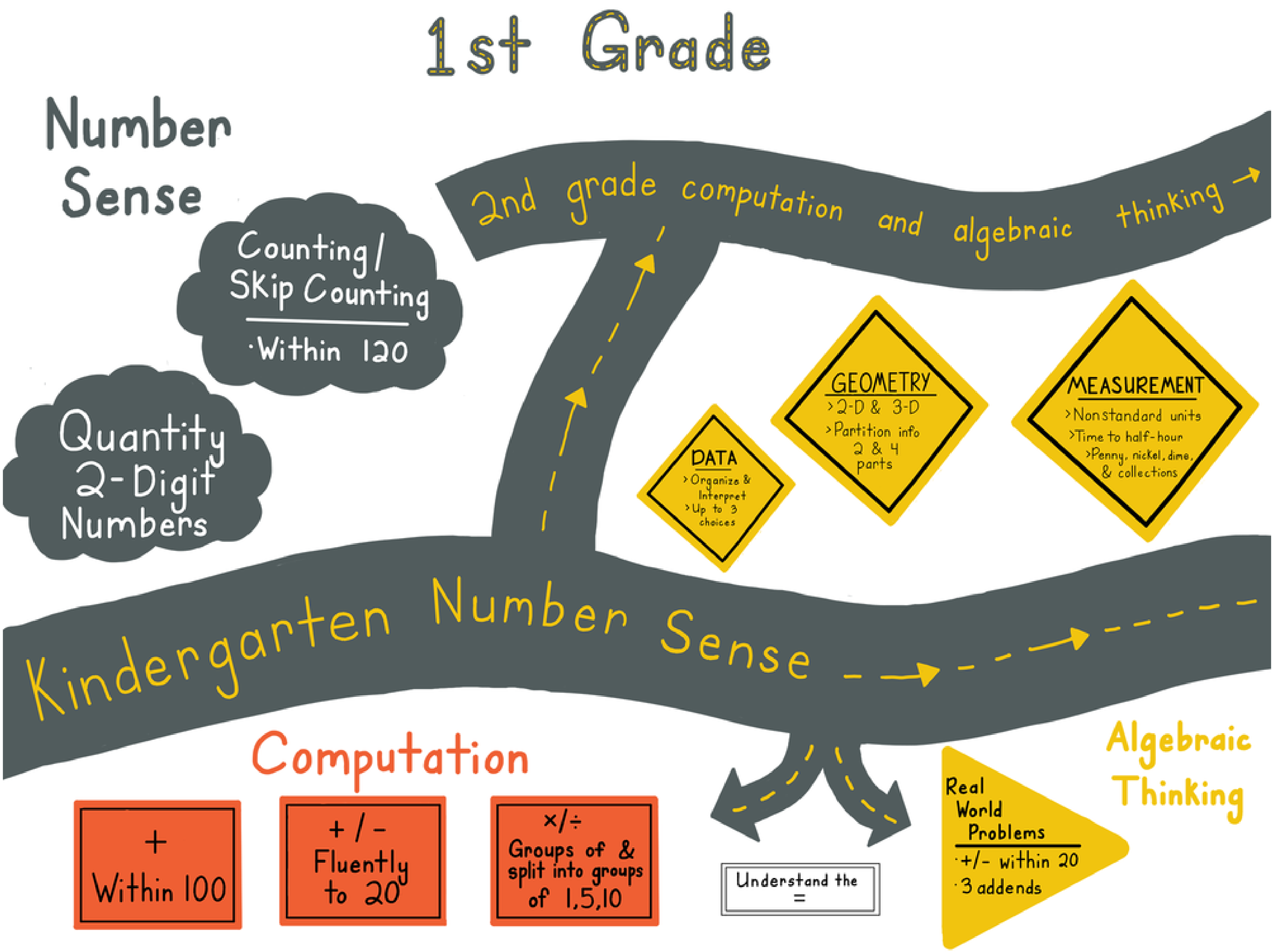
KEEP INDIANA LEARNING

1st Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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1st Grade Big Math Ideas - Narrative

Number Sense

Number sense will be a presence in the first grade classroom across the entire year. Building off the counting-by-1s work done in Kindergarten, first graders will be exploring numbers up to 120. They will learn to count by 1s, 5s, and 10s in first grade. This work should be done with hundreds charts, number lines, and other tools to model thinking. First graders should be given visuals such as those listed, in order to help them to notice and name patterns in skip-counting. Be sure to also use pennies, nickels, and dimes (and collections of pennies, nickels, and dimes) to help teach counting by 1s, 5s, and 10s. Counting these coins and collections is a measurement standard in first grade, so you're supporting two skills at the same time! Teaching should focus on sense-making with these counting patterns, not the memorization of the pattern. Skip-counting is an important precursor to multiplication and division, so spend the time needed on this content. Our Multiplication and Division Progressions (coming soon) will provide you with detailed information about how skip-counting grows across the grades as students move toward formal multiplication and division.

Counting routines should take place daily in classrooms. Make this work visible by recording the counting sequence on the board, number chart, or number line to help students discover the patterns in skip-counting. After recording the counting pattern, ask students to tell you what they are noticing about the patterns in the counts. This work will help students start to make sense of the counting patterns in our counting system. It will also be important to use objects such as unifix cubes and base-ten pieces to help students with their counting work. Students should work on their counting in order to develop a strong understanding of the quantity of a number. By using tools and manipulatives, students will strengthen their initial understandings from Kindergarten as they recognize larger numbers also represent a specific quantity.

In addition to counting and quantity, first graders will be spending much time working to understand the role of ten in our number system. They should be using ten-frames, base-ten pieces, and other tools to represent objects, and be

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encouraged to make interpretations about how the model matches the written numeral, including understanding that each digit represents a number of tens or a number of ones. All year long students should build and model numbers. Again, the goal of this work is to help students truly understand the quantity of a number so they can be flexible with that number/quantity. This flexibility will be important as they generalize understandings to larger numbers and as they work to develop computation strategies. It can be tempting to quickly work through this content by focusing on supporting students to memorize the rules of the place-value system; however, it is incredibly important first graders understand numbers in deep ways.

It will be important to know what students learned in Kindergarten and what they will learn in Second Grade. Be sure to visit our K-2 Number Sense Progressions (coming soon) to see more detailed information about how students grow their knowledge in grades K-2.

Computation and Algebraic Thinking

Students will also spend significant time developing computation and algebraic thinking skills. This content will build off of the work they started in Kindergarten. Some of the biggest work in first grade will be developing fluency with addition and subtraction facts within 20. Know that first grade students were exposed to equations with addition and subtraction up to 10 in Kindergarten, including solving real-world problems. Lean on this knowledge and help students quickly start to apply their within-10 computation skills to numbers within 20. The same strategies which worked in Kindergarten will work again in first grade: using objects, drawings, ten-frames, base-10 pieces, and composing/decomposing. To learn more about the addition and subtraction strategies students learned in Kindergarten, visit our Addition and Subtraction Progressions (coming soon).

Computation and Algebraic Thinking content should be taught through both naked number problems and real-world problems. Students should work with word problems that include an unknown quantity in all parts of the problem: start, change, and result. Starting computation practice with real-world problems will be important, as it will help students develop an understanding of what is actually happening when combining or separating values. Remember, being fluent with facts (and real-world computation) isn't "fast." Fluency means being **flexible**, **efficient,** and

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accurate. Students should work **flexibly** with numbers and strategies and should choose strategies that will **efficiently** help them reach an **accurate** result that they can explain. Also, remember strategy-based approaches to computation and fluency result in students better knowing and retaining their facts. Furthermore, these strategies will benefit them as they move into computing with more complex numbers. This is explained in detail in our Addition and Subtraction progression documents as well as our Fraction Computation document (coming soon). So, support students in using strategies such as counting on, making a ten, decomposing a ten, the relationships between addition and subtraction, and creating easier known problems. Students should spend ALL year on this computation and fluency work. It is important to remember students must develop AND maintain fluency with facts; therefore, students should be given ongoing opportunities to engage with addition and subtraction fluency in engaging ways. This includes warm-ups, math fact games (e.g., Computation Top-It, Salute, Race to 100, etc.), targeted fact practice (choose specific, related facts for students to practice individually and in games), fact interviews that require students to explain their strategies when solving facts, and more. It is important to avoid timed-tests, as they only support students in developing accuracy, which is only one component of fluency and does not support students in long-term retention of math facts as well as strategy-based learning does.

In addition to adding and subtracting within 20, first graders will be adding within 100, specifically adding a two-digit to a multiple of ten and adding a two-digit and a one-digit number. Pay careful attention to the problems students are solving and resist the urge to extend into more complex problems too quickly. Give students time to develop their strategies and to become comfortable with them. Remember, they will extend on this learning in second grade as they work to develop fluency with addition and subtraction within 100. It is important, again, that addition and subtraction instruction and practice focuses on strategies and using number sense skills to solve. Remember, students have been working on counting by 1s, 5s, and 10s, and they can use those skills to solve problems. They should also be using tools such as ten-frames and base-ten pieces to help them to add. First graders should NOT be introduced to the traditional algorithm for addition. This is a fourth grade standard and is an inefficient and developmentally inappropriate strategy with the numbers

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first graders are working with. Instead, students should be using the same strategies they are using when solving basic facts (such as counting on, making a ten, decomposing a ten, the relationships between addition and subtraction, and creating easier known problems) in order to develop flexibility with numbers, application of number sense skills, and efficiency when choosing between strategies. Use ten-frames, number lines, hundreds charts, and manipulatives to support this work all year. To see how this work fits into the progression of learning toward the U.S. Traditional Algorithms, visit our Addition and Subtraction Progressions (coming soon).

Number Sets

It is important to carefully consider the number size being used when supporting specific skills with students. Some number sets will be used to support Number Sense skills, while other sets are used in computation. Here is a summary of the work First Grade students will do with numbers of specific sizes:

Numbers within 120	Count and skip count by 1s, 5, 10s Read and write numerals Represent a set of objects with a numeral
Numbers within 100	Represent numbers using tens and ones Mentally find 10 more/less Add using models and place-value strategies, operations, and relationships between addition and subtraction
Numbers within 20	Understand place value Fluently add and subtract Solve real-world problems involving addition (with up to three addends) and subtraction Create real-world problems to represent equations
Numbers within 10	Understand place value Ordinal numbers

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Geometry

First graders will spend some time learning about 2-D and 3-D shapes and organizing and classifying shapes based on attributes. They will also decompose shapes into smaller shapes and use shapes to compose larger shapes, and they will start to partition squares and circles into 2 and 4 parts. Both of these skills are foundations for their fraction knowledge. As students decompose, compose, and partition, they should start describing the parts as halves and fourths. Visit our Fractions Progression (coming soon) to see more about how this work is connected to fraction learning.

Additional Learning

Other important ideas in first grade fall into the category of measurement. Students should spend time across the year working with non-standard units of measure to learn how to use measurement tools. It is important they spend time with non-standard units of measure because it will help them understand that traditional measurement tools are used to measure a number of increments that represent specific quantities (i.e., they will come to understand that a ruler measures length by finding the number of 1-inch increments).

First graders should also spend time across the year learning about time and practicing telling time with digital (hours and minutes) and analog clocks (to the half hour). Students will also apply their counting and skip-counting skills to find the value of pennies, nickels, dimes, and a collection of pennies, nickels, and dimes. Give students practice with this, leaning on their counting work to build strong understanding.

Students will also collect and organize data with three choices, and ask and answer basic questions about that data. This work can take place in a variety of ways, including warm-ups, cross-content studies, and other math experiences.

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First Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Number Sense	**1.NS.1** Count to at least 120 by ones, fives, and tens from any given number. In this range, read and write numerals and represent a number of objects with a written numeral.	High	Students will understand that numbers follow a sequence and will apply that sequence when counting by 1s, 5s, and 10s up to 120. Students will understand that numbers can be represented by written numerals. Students will understand that a written numeral can be used to represent a quantiy. Students will count to 120 by 1s, 5s, and 10s from any given number up to 120.
	1.NS.2 Understand that 10 can be thought of as a group of ten ones — called a "ten." Understand that the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. Understand that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	High	Students will understand that ten ones can be combined to make one ten. Students will understand that a ten can be decomposed into ten ones. Students will understand that the numbers 11-19 are made up of a ten and some ones. Students will understand that the "tens" (10, 20, 30, 40, 50, 60, 70, 80, 90) refer to a number of tens and no ones. Students will be able to represent numbers as a number of tens and ones.
	1.NS.3 Match the ordinal numbers first, second, third, etc., with an ordered set up to 10 items.	Medium	Students will understand that counting objects follows a sequence. Students will undersand that ordinal numbers are used to name an object's placement in a sequence. Students will be able to count a set of objects up to 10. Students will be able to use ordinal numbers to name an object's place in a sequence.
	1.NS.4 Use place value understanding to compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	Medium	Students will understand that the placement of a digit impacts the value of the digit. Students will understand that in a two-digit number, each digit represents a number of tens or a number of ones. Students will compare two-digit numbers by applying their understanding of the value of digits in a two-digit number. Students will use the comparison symbols to record a comparison. Students will explain and justify their mathematical thinking with models, words, and pictures.

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	1.NS.5 Find mentally ten more or ten less than a given two-digit number without having to count, and explain the thinking process used to get the answer.	High	Students will understand that a change in a digit the tens place results in a change of "ten". Students will model what ten more and ten less looks like using tools like hundreds chart and number lines. Students will be able to find ten more or ten less than a two-digit number by explaining that a change of "1" in the tens place is actually a change of "10", and will be able to mentally find ten more or ten less than a given number without counting.
	1.NS.6 Show equivalent forms of whole numbers as groups of tens and ones, and understand that the individual digits of a two digit number represent amounts of tens and ones.	High	Students will understand that the placement of a digit in a two-digit number tells if the digit represents the number of tens or the number of ones. Students will attend to precision when using the terms "digit" and "number." Students will understand that two-digit whole numbers can be thought of as a number of tens and a number of ones. Students will understand that a "ten" is comprised of one ten or ten ones. Students will be able to show the value of two-digit whole numbers in more than one way, using tens and ones.
	1.CA.1 Demonstrate fluency with addition facts and the corresponding subtraction facts within 20. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a 10 (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). Understand the role of 0 in addition and subtraction.	High	Students will understand that addition is the combining of two or more values. Students will understand that subtraction is finding the difference between two or more values. Students will use strategies such as counting on, making ten, and decomposing numbers to solve addition and subtraction problems. Students will use known facts to solve facts on which they are still working. Students will understand the relationship between addition and subtraction. Students will demonstrate flexibility with strategies and will choose the strategies that works best for them to solve the problem. Students will demonstrate efficiency with their use of strategy and in finding the solution to the problem. Students will demonstrate accuracy with the use of strategies and the solution to the problem. Students will demonstrate fluency (flexibility, efficiency, and accuracy) with addition and subtraction facts within 20.

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	1.CA.2 Solve real-world problems involving addition and subtraction within 20 in situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all parts of the addition or subtraction problem (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).	High	Students will understand that addition is the combining of two or more values. Students will understand that subtraction is finding the difference between two or more values. Students will solve problems in situations of adding to, taking from, putting together, taking apart, and comparing. Students will solve problems in the above situations and will solve for all parts of problem (start, change, result) Students will use strategies such as using objects, drawings, and equations to solve problems. Students will solve addition and subtraction real-world problems using strategies.
	1.CA.3 Create a real-world problem to represent a given equation involving addition and subtraction within 20.	High	Students will understand that addition is the combining of two or more values. Students will understand that subtraction is finding the difference between two or more values. Students will understand that word problems involving addition and subtraction result in the combining or separating of values, or in finding the difference between values. Students will understand that real-world problems are problems that are realistic and relevant to their lives. When given an equation with a missing number, students will generate word problems that require addition or subtraction to solve.
Computation and Algebraic Thinking	**1.CA.4** Solve real-world problems that call for addition of three whole numbers whose sum is within 20 (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).	High	Students will understand that addition is the combining of two or more values. Students will understand that strategies such as using objcts, drawing pictures, and writing equations can be used to represent and solve addition problems. Students will be able to solve addition problems within 20 with 3 addends using multiple stratgies.

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	1.CA.5 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; describe the strategy and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and that sometimes it is necessary to compose a ten.	High	Students will understand that addition is the combining of two or more values. Students will understand how to use models that include but are not limited to number lines, arrow language, partial sums, and base ten blocks to represent addition problems. Students will use drawings to represent addition problems. Students will understand place value of two-digit numbers. Students will understand that when using place value to add, they will combine the tens with tens and ones with ones. Students will understand that a ten is composed of ten ones, and that sometimes, in addition, they must compose a new ten. Students will understand the properties of addition: property of zero, commutative property). Students will understand the relationships between addition and subtraction. When solving problems involving addition within 100, and in the addition of a two-digit and a one-digit number and a two-digit number and a multiple of ten, students will use above strategies and understandings to solve.
	1.CA.6 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2).	High	Students will understand the the equal sign is a symbol used to show balance or equivalence. Students will understand that the equal sign does NOT mean that the answer will follow. Students will understand that a math equation can be true or false. Students will use strategies to determine the value of an expression on each side fo the equal sign. Students will determine if statements are true or false by using relational and/or computational thinking.
	1.CA.7 Create, extend, and give an appropriate rule for number patterns using addition within 100.	Medium	Students will understand that patterns are built from repeated changes. Students will understand that patterns can be created or found. Students will know that patterns with numbers can be creating using addition to move from one number to the next. Students will create addition patterns using numbers within 100. Students will extend a given addition pattern within 100. Students will give addition rules for patterns within 100.

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Geometry	**1.G.1** Identify objects as two-dimensional or three-dimensional. Classify and sort two-dimensional and three-dimensional objects by shape, size, roundness and other attributes. Describe how two-dimensional shapes make up the faces of three-dimensional objects.	Medium	Students will understand the difference between two-dimensional and three-dimensional shapes. Students will understand that to "classify" or "sort" means to create groups of objects that share specific characteristics or attributes. Students will understand that shapes can be characterized by their size, shape, roundness, and other attributes. Students will recognize that the faces of three-dimensional shapes are two-dimentional shapes. Students will be able to sort, classify, and descibe two-dimensional and three-dimensional shapes, including telling the similarities and differences between the shapes in each group. Students will be able to explain their thinking when sorting and classifying.
	1.G.2 Distinguish between defining attributes of two- and three-dimensional shapes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size). Create and draw two-dimensional shapes with defining attributes.	Medium	Students will understand that shapes are classified based on specific characteristics, including number of size, number and size of corners/verticies, etc. Students will understand that other characteristics do not define the shape: color, orientation, overall size, etc. Students will be able to create and draw two-dimensional shapes using their understanding of the defining attributes of the shape.
	1.G.3 Use two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or threedimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. [In grade 1, students do not need to learn formal names such as "right rectangular prism."]	Medium	Students will understand that the faces of three-dimensional shapes are two-dimensional shapes. Students will be able to tell the characteristics of two-dimensional shapes such as rectangles, squares, trapeziods, triangles, half-circles, and quarter-circles. Students will be able to tell the attributes of cubes, right rectangular prisms, right circular cones, and right circular cylinders (but do not need to know the formal names). Students will be able to compose and decomponse with two-dimensional or three-dimensional shapes by createing composite shapes and also using the composite shape to create new shapes.

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	1.G.4 Partition circles and rectangles into two and four equal parts; describe the parts using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of, the parts. Understand for partitioning circles and rectangles into two and four equal parts that decomposing into equal parts creates smaller parts.	High	Students will understand that "equal parts" are parts that have the same size. Students will understand that to "partition" means to decompose a shape into smaller, equal parts. Students will understand that the number of equal parts a shape is partitioned into will determine the size and name of the parts. Students will understand that the "whole" shape is comprised of two of (or four of) those parts. Students will be able to partition shapes into two and four equal parts. Students will be able to use words and phrases such as half, halves, fourth, fourths, half of, and quarter of to descibe the parts.
Measurement	**1.M.1** Use direct comparison or a nonstandard unit to compare and order objects according to length, area, capacity, weight, and temperature.	Medium	Students will understand that objects can be compared according to length, area, capacity, weight, and temperature. Students will understand that comparisons can be made through direct comparison or by using nonstandard units of measure. Students will compare and order objects using direct comparison or nonstandard measures.
	1.M.2 Tell and write time to the nearest half-hour and relate time to events (before/after, shorter/longer) using analog clocks. Understand how to read hours and minutes using digital clocks.	Medium	Students will understand that time is a measurement that is used to organize days, weeks, months, years. Students will understand that the words before/after, shorter/longer can be used to describe the length of events. Students will be understand that hours can be broken apart into half-hours. Students will understand that digital and analog clocks both measure time. Students will be able to read an analog clock to tell time to the nearest half-hour. Students will be able to write the time from an analog clock (to the half-hour). Students will be able to read hours and minutes on a digital clock.
	1.M.3 Identify the value of a penny, nickel, dime, and a collection of pennies, nickels, and dimes.	High	Students will understand that our money system includes coins that have different values. Students will understand that coins can be combined to create larger values. Students will understand that a single coin has one value only. Students will be able to tell the value of a penny, nickel, and dime. Students will be able to combine the values of pennies, nickels, and dimes in a collection (using addition strategies).

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Data Analysis	**1.DA.1** Organize and interpret data with up to three choices (What is your favorite fruit? apples, bananas, oranges); ask and answer questions about the total number of data points, how many in each choice, and how many more or less in one choice compared to another.	Medium	Students will understand that information can be organized as data. Students will understand that information can be collected through the asking and answering of questions. Students will be able to collect and organize data. Students will be able to answer questions about organized data, such as finding the total number of data point, comparing one choice to another, or telling how many more of one choice compared to another.

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Big Math Ideas

2nd Grade

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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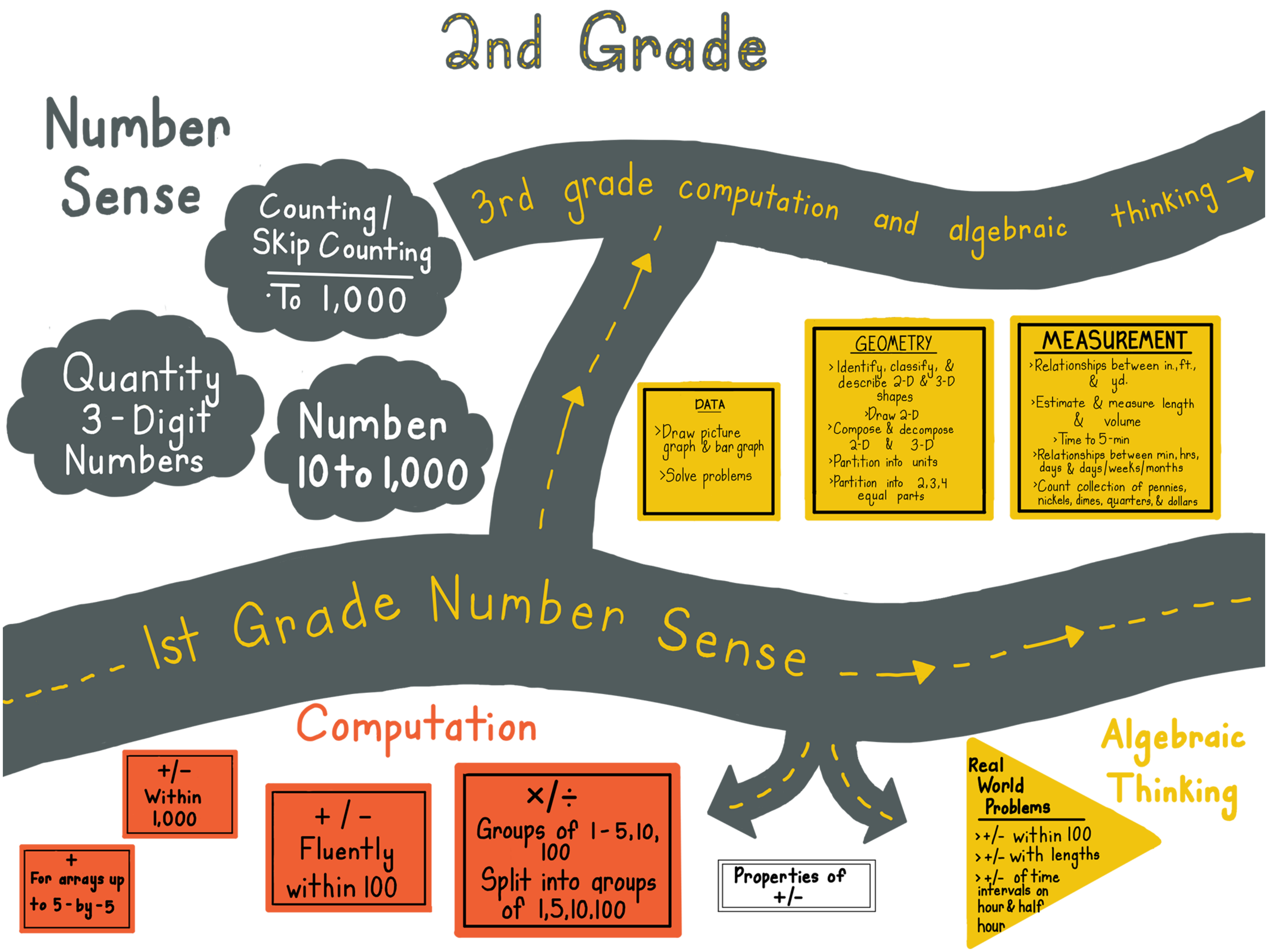
2nd Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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2nd Grade Big Math Ideas - Narrative

Computation and Algebraic Thinking

Some of the biggest work in second grade will be their computation work. Students will be using all the number sense and strategy work from Kindergarten and first grade to solve larger problems. Students will be developing fluency with addition and subtraction within 100 by solving naked number problems and real-world problems in situations of adding to, taking from, putting together, taking apart, and comparing (with unknowns in all parts of the problem: start, change, and result). In first grade, students worked to solve these types of problems using strategies such as place-value strategies and properties of operations. Be ready to build off of this work in second grade in order to help students move toward fluency with this computation. Remember, fluency means being **flexible**, **efficient**, and **accurate** with strategy choice and use. Students should work **flexibly** with numbers and strategies and should choose strategies that will **efficiently** help them reach an **accurate** result that they can explain. It is important they are supported in developing many strategies that lean on their number sense and computation skills. Please note that first grade standards are heavy on addition. They focus on adding within 100 - not subtracting. You will want to start your second grade year with subtraction concepts and give them a chance to build their conceptual understanding of subtraction as they did with addition in first grade. Give students time and space to explore and develop their own strategies and to practice the strategies introduced to them. Keep in mind this does not include the U.S. traditional subtraction algorithm. This is a fourth grade standard and should NOT be taught in second grade. Strategies should be based on number sense and place value skills.

Second graders will also be developing strategies to add and subtract within 1,000. They will be composing and decomposing numbers to create problems that are easier to solve, using drawings and objects, and using strategies such as breaking apart a number by its place values. It is very important students are also able to describe their strategy, including how they used it and why they selected that strategy. They should also be asked to determine if their answer is reasonable, using

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estimation strategies to check for reasonableness. At this age, students are starting to form generalizations about why specific strategies work for specific types of problems, and the opportunity to explain their thinking will be important for these generalizations to develop. This computation should be supported across the entire year, with more and more complex problems as the year moves along. Giving students time and space to develop and practice strategies and to apply them to more complex problems will ensure they have a strong understanding of the computation work they are doing. When they get to third grade, they will have more opportunities to develop fluency with their computation within 1,000; so, for now, give students time to work on their strategies within 100. Please remember that just because they are adding and subtracting within 1,000, that doesn't mean they are using the standard US algorithm. Please see our Addition and Subtraction Progression documents (coming soon) to help you better understand the reasoning behind this.

It is also important to note that second grade students will require much time to continue to work with their basic math facts. Students in first grade started to develop this fluency, but remember that fluency must be retained. Students will need to continue to have experience, exposure, and time with addition and subtraction facts. Also remember that strategy-based approaches to computation and fluency result in students better knowing and retaining their facts. So, support students in using strategies such as counting on, making a ten, decomposing a ten, the relationships between addition and subtraction, and creating easier known problems when working with math facts. Your goal is to help them reach automaticity with these facts, not to make them memorize the facts. It is important to avoid timed-tests, as they only support students in developing accuracy, which is only one component of fluency and does not support students in long-term retention of facts. Instead, warm-ups, math fact games (Computation Top-It, Salute, Race to 100, etc.), targeted fact practice (choose specific, related facts for students to practice individually and in games), fact interviews that require students to explain their strategies when solving facts, and other sense-making activities should be utilized across the year.

Second grade students should NOT be introduced to the traditional algorithm for addition or subtraction. The traditional addition and subtraction algorithms fall in

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fourth grade standards. These algorithms are inefficient strategies for the problems second graders are solving. The algorithms also detract from the important strategy and number sense work second graders should be doing. When helping students to solve multi digit problems that require a type of regrouping, we should help them understand they are renaming the number. This is when their flexibility and deep understanding of numbers will support them. When solving problems involving addition, it is important they understand they are composing a new group of a different size (ten ones=10, not "carrying a 1". It is also important they understand that regrouping in subtraction is decomposing to a different size (one ten=ten ones; not "borrowing"). For more information on this topic, see the Addition and Subtraction Progressions (coming soon). The computation work in second grade should provide ongoing support with developing and using strategies. Across the year, support students in moving toward more and more efficient and sophisticated strategies.

Number Sense

Another big idea in second grade is supporting the number sense work up to 1,000. Students will be counting and skip counting up to 1,000, reading and writing numbers up to 1,000, and using models, standard form, and expanded form to show numbers up to 1,000. This work helps students to continue building on their understanding of quantity and to develop flexible ways to think about numbers. This work should be done using base-ten pieces, place value charts, number lines, drawings, and other tools. For more information about the progression of Number Sense skills in grades K-2, check out our K-2 Number Sense Progressions (coming soon).

One second grade skill that is sometimes overlooked is understanding how to determine if a number is even or odd. It is important students practice strategies and methods for determining if a number is even or odd. They should practice separating objects into equal groups and lining up to compare. Remember to emphasize that an even number can be divided into equal whole-number groups. Consider giving students objects to sort and compare. Beyond determining if a number is even or odd, this understanding is supportive of the multiplication and division work they will be doing in later grades. Resist the temptation to teach students to only look at the

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final digit to determine if it is even or odd, as this will not reach the goal of developing foundational understandings of division. Visit our Multiplication and Division Progressions (coming soon) to learn more.

Number Sets

It is important to carefully consider the number size used when supporting specific skills with students. Some number sets will be used to support Number Sense skills, while other sets are used in computation. Here is a summary of the work second grade students will do with numbers of specific sizes:

Numbers to at least 1,000	count by 1s, 2s, 5s, 10s, and 100s from any given number
Numbers up to 1,000	read and write whole numbers show equivalent forms of whole numbers compare numbers on a number line understand place values (100s, 10s, 1s,) understand "1 hundred" is the same as 10 tens use place-value to compare 3-digit numbers add and subtract using drawings and strategies addition and subtraction patterns
Numbers within 100	add and subtract fluently solve real-world problems involving addition and subtraction
Numbers up to 30	ordinal numbers
Numbers up to 25	use addition to find the total number in an array
Numbers up to 20	even and odd sets of objects

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Geometry

Second grade students will also be introduced to area, multiplication, and fractions, all through the use of partitioning shapes. They should spend some time on this work, discovering ways they can partition squares, rectangles, and circles and learning to name the parts they create. They area also asked to use terms like halves, thirds, and fourths, but they do not need to label their partitioned shapes as such. Leave that for 3rd grade work, and spend time helping them understand what partitioning is and attending to precision with the use of terms in your classroom. This work provides a foundation and introduction to fractions, which will be built on in third grade. Visit the Fractions Progression (coming soon) to see more about how this content is connected.

Students should also start to explore using rows and columns to partition a square or rectangle, and recognize they can use repeated addition or multiplication to find the total number of square units without counting. This work builds on Kindergarten and first grade foundational multiplication skills, but introduces a new representation of multiplication through the use of arrays. Spending time helping students learn this content will set them up for success as they enter third grade. Again, use drawings, manipulatives, and objects to help students understand this new learning, and visit the Multiplication Progressions (coming soon) to learn more about the vertical articulation of these skills.

Time

Students should also be using computation skills with time and data problems. Students will learn to tell time to the nearest five-minutes. Students should also work with word problems using intervals to the hour and half hour in addition and subtraction problems. Keep in mind that students should work to solve these types of problems by using strategies and models.

Additional Learning

Students in second grade will also continue to learn about 2-D and 3-D shapes, including how to construct them. They will spend time learning how to use

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standard units of measure and to consider how the length of the unit affects the number of units needed to represent the length of an object. Students also need to create single-scale bar graphs and picture graphs to represent data with up to four choices. Students should also be able to solve problems using the data, including put-together, take-apart, and compare problems, leaning on computation strategies to solve.

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Second Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
	2.NS.1 Count by ones, twos, fives, tens, and hundreds up to at least 1,000 from any given number.	High	Students will understand that our number system follows a sequence. Students will understand that skip counting follows a sequence. Students will count by 1s, 2s, 5s, 10s, and 100s up to at least 1,000. Students will count from any given number by 1s, 2s, 5s, 10s, and 100s up to at least 1,000 Students will explain how they know what numbers comes next in the skip counting sequence.
	2.NS.2 Read and write whole numbers up to 1,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 1,000.	High	Students will understand that whole numbers can be represented in different ways. Students will understand that digits in a number have value, and that value is determined by the digits placement in the number. Students will understand that numbers can be decomposed in more than one way, including by adding the value of each digit. Students will write whole numbers in word form. Students will write whole numbers using standard form. Students will write whole numbers in expanded form.
	2.NS.3 Plot and compare whole numbers up to 1,000 on a number line.	Medium	Students will understand that numbers represent a quantity. Students will understand that a "larger" number represents a larger quantity. Students will understand that a number line can be used to plot numbers in order. Students will plot numbers on a number line. Students will use plotted numbers on a number line to compare the numbers. Students will explain and justify their mathematical thinking with models, words, and pictures.

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Number Sense	**2.NS.4** Match the ordinal numbers first, second, third, etc., with an ordered set up to 30 items.	Low	Students will understand that counting follows a sequence. Students will undersatnd that a set of objects can be counted and that each number said matches exactly one object. Students will know the ordinal words for objects up to 30. Students will count up to 30 items, using one number name for each object. Students will use ordinal words to describe an object's placement in a sequence.
	2.NS.5 Determine whether a group of objects (up to 20) has an odd or even number of members (e.g., by placing that number of objects in two groups of the same size and recognizing that for even numbers no object will be left over and for odd numbers one object will be left over, or by pairing objects or counting them by 2s).	High	Students will understand that an even number is a number whose quantity can be divided into two equal whole-number groups. Students will understand that an odd number is a number whose quantity cannot be divided evenly into two equal whole-number groups. Students will develop and explain strategies to determine if a quantity up to 20 is even or odd, such as lining up and matching, placing objects into groups of the same size, pairing objects, and counting by 2s.
	2.NS.6 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones). Understand that 100 can be thought of as a group of ten tens - called a “hundred.” Understand that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	High	Students will understand that the digits in a three-digit number represent a number of hundreds, tens, and ones. Students will understand that the placement of a digit in a three-digit number determines the value of that digit. Students will understand that 100 can be thought of as ten "tens" or one hundred "ones". Students will understand that the "hundreds" (100, 200, 300, 400, 500, 600, 700, 800, 900) represent a number of hundreds, zero tens, and zero ones. Students will interpret three-digit numbers by explaining the value of each digit in the number according to its place value.

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	2.NS.7 Use place value understanding to compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	Medium	Students will understand that numbers represent a quantity. Students will understand that numbers can be compared based on the quantity they represent. Students will understand that comparing three-digit numbers means examining each place value to determine which number has the digits that represent the largest value. Students will compare two three-digit numbers by explaining the value of each digit and how those values relate to the quantity they represent. Students will use the <, >, = symbols to complete comparisons between two three-digit numbers. Students will use words and pictures to justify their mathematical thinking when comparing two three-digit numbers.
	2.CA.1 Add and subtract fluently within 100.	High	Students will use flexibility, accuracy, and efficiency in determining the best strategy for solving addition and subtraction problems within 100. Students will understand that there are many strategies that can be used for addition and subtraction. Students will use partial sums, partial differences, base-ten models, number lines, etc. to add and subtract fluently within 100.
	2.CA.2 Solve real-world problems involving addition and subtraction within 100 in situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all parts of the addition or subtraction problem (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem). Use estimation to decide whether answers are reasonable in addition problems.	High	Students will engage in 11 different problem types. Students will write an equation that matches the real world problem. Students will identify the meaning of each number in the equation. Students will use strategies that include, but are not limited to number lines, arrow language, base-ten blocks, and partial sums to solve real world problems. Students will solve problems using more than one strategy. Students will describe the strategies they use.

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Computation and Algebraic Thinking	**2.CA.3** Solve real-world problems involving addition and subtraction within 100 in situations involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers, and equations with a symbol for the unknown number to represent the problem).	High	Students will solve story problems involving lengths by using strategies that include but are not limited to drawings, number lines, and partial sums. Students will solve problems using more than one strategy. Students will describe the strategies they use. Students will write equations that match the story problem. Students will identify the meaning of each number in the equation.
	2.CA.4 Add and subtract within 1000, using models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; describe the strategy and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones, and that sometimes it is necessary to compose or decompose tens or hundreds.	High	Student will add and subtract numbers within 1,000. Students will use strategies that include, but are not limited to number lines, arrow language, base-ten blocks, and partial sums. Students will use more than one strategy to solve the problem. Students will explain their strategies through oral and written language. Students will add hundreds to hundreds, tens to tens, ones to ones, and know when they need to compose or decompose numbers. They will be able to explain their reasoning and represent all thinking on paper through words, pictures, and/or other models. Students will subtract hundreds from hundreds, tens from tens, and ones from ones. In doing so, they will determine when they need to compose or decompose numbers to successfully subtract. They will be able to explain their reasoning and represent all thinking on paper through words, pictures, and/or other models.
	2.CA.5 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal groups.	High	Students will organize a collection of objects as arrays up to 5 x 5. Students will write an addition problem that matches the array. Students will orally use the langaguage "groups of" when describing the design of the array.
	2.CA.6 Show that the order in which two numbers are added (commutative property) and how the numbers are grouped in addition (associative property) will not change the sum. These properties can be used to show that numbers can be added in any order.	Low	Students will understand the meaning of the equal sign. Students will create and draw addition expressions. Students will explain orally and through the written word how they commutative and associative property works.

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	2.CA.7 Create, extend, and give an appropriate rule for number patterns using addition and subtraction within 1000.	Medium	Students will understand that patterns are created of a repeating change. Students will understand that skip-counting is a repeating change. Students will describe patterns in addition and subtraction. Students will explain patterns when adjusting addends (46 + 98 is the same as 44 + 100).
Geometry	**2.G.1** Identify, describe, and classify two- and three-dimensional shapes (triangle, square, rectangle, cube, right rectangular prism) according to the number and shape of faces and the number of sides and/or vertices. Draw two-dimensional shapes.	Medium	Students will understand that shapes can be named and classfied according to attributes such as number and shape of faces, number of sides and/or verticies Students will know how to count the number of faces and sides and/or verticies. Students will be able to use the faces, sides, and verticies to identfiy, describe, and classify two- and three-dimensional shapes. Students will be able to draw two-dimensional shapes.
	2.G.2 Create squares, rectangles, triangles, cubes, and right rectangular prisms using appropriate materials.	High	Students will describe the attributes of squares, rectangles, triangles, and right rectangular prisms. Students will create squares, rectangles, triangles, and right rectangular prisms.
	2.G.3 Investigate and predict the result of composing and decomposing two- and three-dimensional shapes.	Medium	Students will understand that shapes can be composed and decomposed. Students will decompose two-dimensional and three-dimensional shapes and describe the resulting parts. Students will compose two-dimensional and three-dimensional shapes based on their attributes.
	2.G.4 Partition a rectangle into rows and columns of same-size (unit) squares and count to find the total number of same-size squares.	High	Students will understand that rectangles can be partioned into numbers of rows and numbers of columns. Students will understand that partitioning shapes means splitting them into smaller, equal parts. Students will be able to parition a rectangle and count the total number of sqaure units.

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	2.G.5 Partition circles and rectangles into two, three, or four equal parts; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, four fourths. Recognize that equal parts of identical wholes need not have the same shape.	High	Students will understand that partitioning is splitting shapes into smaller, equal parts. Students will understand that the parts resulting from partitioning shapes into 2, 3, or 4 parts can be described as halves, thirds, half of, a third of, etc. Students will understand that, when partitioned, the whole shape is two halves, three thirds, or four fourths. Students will understand that shapes can be partitioned into equal parts in more than one way. Students will partition shapes in more than one way, describe each part, and describe the whole as the total number of parts.
	2.M.1 Describe the relationships among inch, foot, and yard. Describe the relationship between centimeter and meter.	Low	Students will understand that length can be measured in different units. Students will understand that length can be measured using inch, foot, and yard. Students will understand that length can be measured in centimeter and meter. Students will understand that there are 12 inches a foot and 3 feet in a yard. Students will understand that there are 100 centimeters in a meter. Students will explain the relationships between inch/foot/yard and centimeter/meter.
	2.M.2 Estimate and measure the length of an object by selecting and using appropriate tools, such as rulers, yardsticks, meter sticks, and measuring tapes to the nearest inch, foot, yard, centimeter and meter.	High	Students will understand that specific tools are used to measure lengths. Students will understand that the length of an object helps to determine which tool to use. Students will understand that the tool being used to measure represents a number of a certain kind of unit. Students will understand that to estimate means to determine a value that makes sense or is reasonable. Students will use measurement tools to measure lengths to the nearest unit.

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Measurement	**2.M.3** Understand that the length of an object does not change regardless of the units used. Measure the length of an object twice using length units of different lengths for the two measurements. Describe how the two measurements relate to the size of the unit chosen.	Medium	Students will understand that objects can be measured using different units. Students will understand that an object does not change lengths when a different unit is used. Students will understand how to measure an object by lining up "zero" and counting the number of units. Students will measure an object more than once and using different units. Students will describe how the number of units is affected by the size of the units (i.e., when measuring and object, the number of centimeters will be greater than the number of inches because centimeters are smaller than inches.
	2.M.4 Estimate and measure volume (capacity) using cups and pints.	Medium	Students will understand that volume/capacity can be measured in cups and pints. Students will use cups and pints to estimate and measure volume/capacity.
	2.M.5 Tell and write time to the nearest five minutes from analog clocks, using a.m. and p.m. Solve real-world problems involving addition and subtraction of time intervals on the hour or half hour.	High	Students will understand that clocks are used to measure amounts of time. Students will understand that a.m. refers to morning hours and p.m. refers to afternoon and evening hours. Students will understand that analog clocks shows minutes and hours. Students will understand that the numbers on a clock tell the hour AND represents 5 minute intervals. Students will tell time to the nearest 5 minutes using analog clocks. Students will solve addition and subtraction problems using times on the hour or half hour.
	2.M.6 Describe relationships of time, including: seconds in a minute; minutes in an hour; hours in a day; days in a week; and days, weeks, and months in a year.	Medium	Students will understand that there are 60 seconds in one minute and 60 minutes in one hour. Students will understand that there are 24 hours in one day, 7 days in one week, and 365 days in most years. Students will understand that there are 52 weeks in one year and 12 months in one year.

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	2.M.7 Find the value of a collection of pennies, nickels, dimes, quarters and dollars.	High	Students will know the value of a penny, a nickel, a dime, a quarter, and a dollar. Students will understand that finding the value of a collection of pennies, nickles, dimes, quarters, and dollars means to combine (add) the values of each part. Students will be able to find the value of a collection of pennies, nickels, dimes, quarters, dollars. Students will use strategies such as skip-counting, counting on, drawing pictures, using models, etc. to find the total value of a collection of coins and dollars.
Data Analysis	**2.DA.1** Draw a picture graph (with single-unit scale) and a bar graph (with single-unit scale) to represent a data set with up to four choices (What is your favorite color? red, blue, yellow, green). Solve simple put-together, take-apart, and compare problems using information presented in the graphs.	Medium	Students will understand that data is a collection of information, and that bar graphs and picture graphs are used to represent a number of choices or votes. Students will understand that a picture graph or bar graph can be used to represent data. Students will draw picture graphs and bar graphs to show data with up to four choices. Students will use the information in bar graphs and picture graphs to solve problems (put-together, take-apart, and compare).

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Big Math Ideas

3rd Grade

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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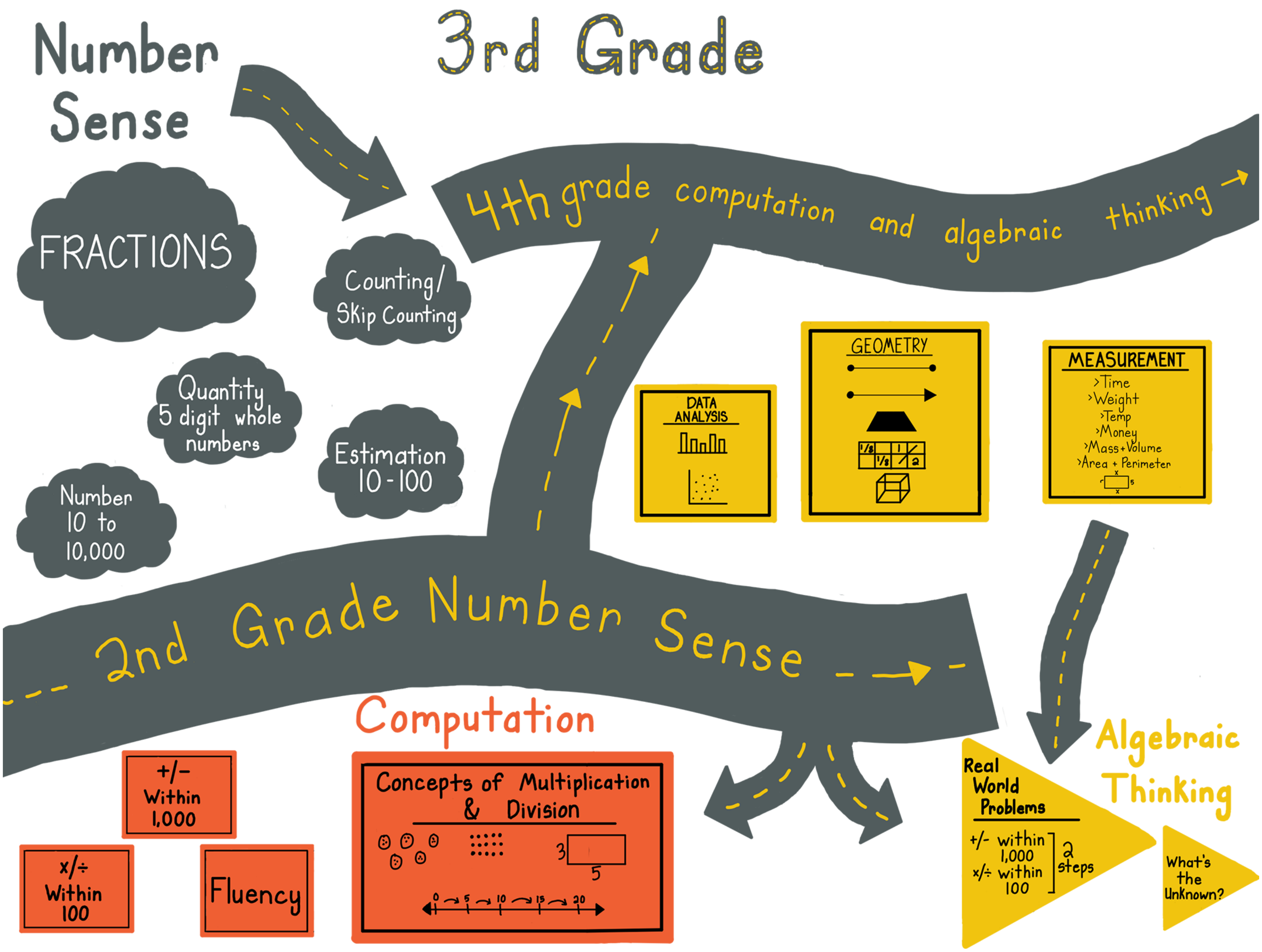
3rd Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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3rd Grade Big Math Ideas - Narrative

Children arrive in their grade equipped with the skills necessary to delve into adding and subtracting larger numbers, developing their multiplication and division skills and beginning work on their understanding of fractions and its notation. Now that they're in third grade and multiplying and dividing, they will also be using all four operations in real-world problems. Not only that, they will use multi-step problems to solve real-world problems involving money and time, and solve real-world problems about area and perimeter. Through all of this real-world problem solving, students are working on the skill of Process Standard 1: Make sense of problems and persevere in solving them.

The following big ideas are what educators should spend the most time on in third grade. This does not mean standards not mentioned are not important. Rather, this is a guide to help teachers see the importance of taking the time needed to develop these concepts before students move on to concepts in fourth and fifth grades.

Multiplication and Division

Let's start with multiplication and division. In third grade, students should spend the entire year working on understanding the meaning of multiplication and division as they develop fluency of facts from 0 x 1 to 100÷10. Fluency does not mean "memorize" or quickly regurgitate facts. Rather, fluency means one must be flexible, accurate, and efficient in solving problems. Students need to be able to **flexibly** think about ways in which they might multiply and divide. They must be able to **accurately** describe how they arrived at the correct answer, and they need to choose the strategy for arriving at the answer in the most **efficient** way. With that clarified, this is not something that can be mastered in a 4 - 6 week "multiplication" and/or "division" unit. Children must have the opportunity to practice multiplication and division on a daily basis. 10 - 15 minutes per day should be used specifically for development of multiplication and division fluency.

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Fluency practice does not mean timed tests. Students can play games and use math talk to explain their computation strategies to their partners; they can look at a series of facts and determine the most efficient way to arrive at the correct answer and explain their strategies through words, pictures, and discussion. Use resources such as *Figuring Out Fact Fluency* by Jennifer Bay-Williams and John SanGiovanni and *Mastering the Basic Math Facts in Multiplication and Division: Strategies, Activities, and Interventions to Move Students Beyond Memorization* by Susan O'Connell to help with basic fact fluency instruction. Build this integral practice time into the daily routine, and make it a sacred time in the classroom.

Before students can begin this fluency practice, however, they must know what multiplication and division actually are. They come to third grade armed with skip counting skills and creating arrays. They have been introduced to the phrases "groups of" and "split into groups of." However, now it's time to apply these ideas to mathematical symbols and real world examples.

In multiplication, students need to understand when reading 5 x 7, one means 5 groups of 7. When reading 7 x 5, one means 7 groups of 5. While the same answer is true of both expressions, their meaning is different. Let's look at these two real world problems:

Deandre has 5 books. Each book has 7 chapters in it. How many chapters will Deandre read if he reads all 5 books?

Deandre has 7 books. Each book has 5 chapters in it. How many chapters will Deandre read if he reads all 7 books?

As adults, we think this is not a big difference; however, a child embarking on reading 5 books might seem less daunting than 7 regardless of how many chapters are in the book. A child drawing a picture model of these two also looks quite different:

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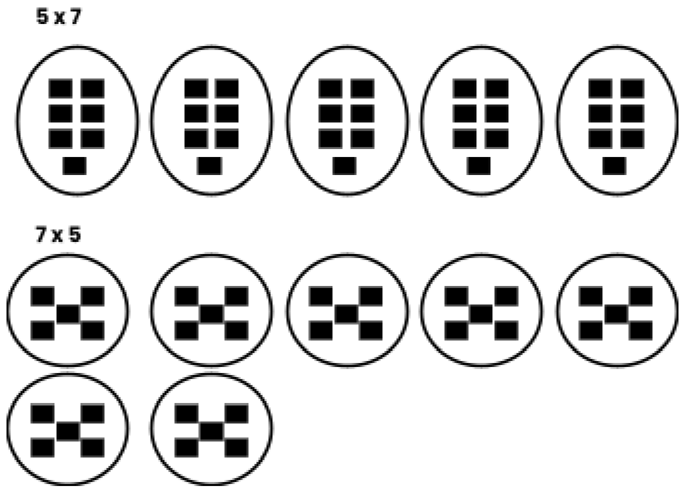
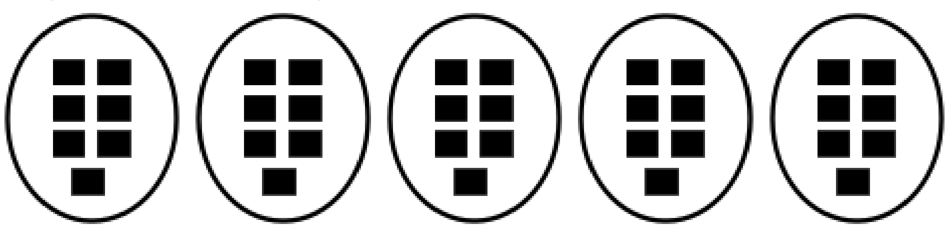


Division must also be taught while attending to precision with its meaning. Division can be equal sharing or partitive. Let's look at 35 ÷ 5:

Sharing:
35 split into 5 equal groups
Real world problem:
My book is 35 pages long. There are 5 chapters. If each chapter is the same length, how many pages are in each chapter?



Each chapter is 7 pages.

Partitive:
35 split into groups of 5
My book is 35 pages long. If each chapter is 5 pages long, how many chapters are in my book?

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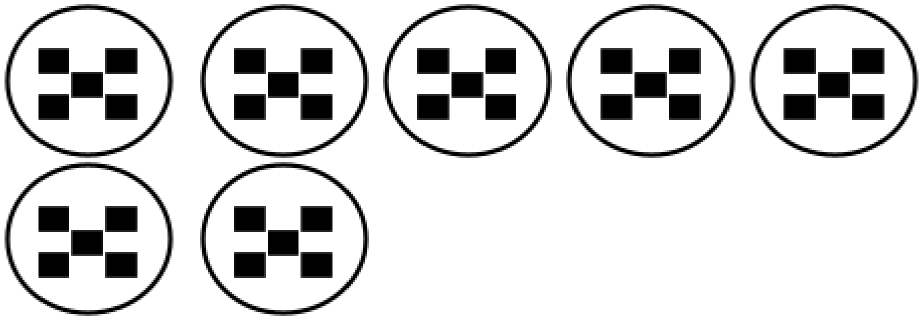

There are 7 chapters in the book.

When looking at these depictions, one can begin to see the inverse relationship that is multiplication and division. Division is not “the opposite of multiplication.” However, there is a direct relationship, and it is imperative children understand this relationship as they begin to make sense of the concepts of multiplication and division.

Multiplication and division do not end with the basic understanding and fluency of basic facts! Students are also asked to solve real-world problems within 100. They can use many different strategies to do so that include, but are not limited to, skip counting, number lines, area models, ratio tables, repeated addition, and repeated subtraction. Be sure to look at our Multiplication Progression and Division Progression (coming soon) for more information on these concepts as well as to understand the concepts children learn in previous grades and why spending this time on these skills in third grade is so important as children move into fourth and fifth grades.

Addition and Subtraction

Students come to third grade being fluent in their basic addition and subtraction facts (first grade) as well as adding and subtracting fluently within 100 (second grade). They have developed multiple strategies to add and subtract these numbers and are ready to apply these skills to adding and subtracting within 1,000. Please note - the traditional U.S. algorithm is not required until fourth grade. Please read our Addition and Subtraction Progressions (coming soon) to understand the many ways in which children should be thinking about numbers and adding and

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subtracting them prior to learning the traditional U.S. algorithm. There is simply no reason to teach this algorithm prior to fourth grade. Spend time honing their skills using various models such as number lines, base ten blocks, arrow language, and partial sums to and leave the traditional algorithm to fourth grade teachers. This extra time you find in NOT teaching the traditional algorithm can be spent on other big ideas such as multiplication and division - AND applying skills to real-world contexts.

Real World Problem Solving

In third grade, children are responsible to solve real-world problems in the following ways:

- Adding within 1,000
- Subtracting within 1,000
- Multiplying within 100
- Dividing within 100
- Compare problems using data found from various forms of graphs and with several categories
- Situations involving money
- Situations involving time
- Situations involving area
- Situations involving perimeter

***Any of these problem types can be involved in two-step story problems.*

In order for students to truly understand real-world problem solving (and also work on the Process Standard 1 (PS 1): Make Sense of Problems and Persevere in Solving Them), they must be regularly given opportunities to solve real-world problems. A few things to consider:

1) Do not rely only on the textbook when assigning real-world problems. Often textbooks have the exact same problem type over and over again. Students should be solving problems where a number is missing from all parts of the equation:

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- 4 + 5 = ___
- 4 + ___ = 9
- ___ + 5 = 9
- 9 - 5 = ___
- 9 - ___ = 4
- ___ - 5 = 4
- 5 x 7 = ___
- 5 x ___ = 35
- ___ x 7 = 35
- 35 ÷ 5 = ___
- 35 ÷ ____ = 7
- ____÷ 5 = 7

2) Students should be working through real-world problems at least 3 times a week. They should be provided time to work together, discuss strategies and outcomes, and share their thinking about how they arrived at the correct answer. Keep in mind, you don't have to teach a problem solving lesson each time you want your students to solve real-world problems. Instead, this practice should be consistent and ongoing so that students can develop strategies over time."; Give them 10 - 15 minutes, 3 days a week to solve problems! Sometimes they'll be connected to a specific lesson on problem-solving, sometimes they won't!
3) Make sure they are using multiple strategies to solve their problems. All problem types can use models including but not limited to pictures, number lines, arrow language, and base ten representation. They should be able to explain all of their thinking and their entire process at arriving at their answers.
4) Stay away from "tips and tricks" and "key words." These do not help children with the PS 1 and impedes their ability to connect their work in third grade to grade levels beyond. Follow these steps:
   a) Read the problem.
   b) What's happening in the problem? What information do you have?
   c) What question do you need to answer?
   d) Look at the problem and determine the information needed to answer the question.

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e) Solve.

f) Is your answer reasonable? If not, what do you need to rethink?

These steps will work for all problem types, every time. Give children the opportunity to become critically thinking problem solvers. This will do nothing but help them as their learning in math continues.

5) Provide ample opportunities for students to write equations that represent problems they are solving. (e.g., On Thursday we planted 27 bags of seed. On Friday, we planted some more. We planted 96 bags of seeds. How many bags of seeds did we plant altogether? 27 + ____ = 96.) Students may use subtraction to *solve* the problem, but that is different from the equation that *represents* the problem. This notion is throughout the standards and important to always include when giving story problem tasks.

Fractions

The last, but certainly not least, big idea in third grade is fractions. Prior to third grade, students have only worked on partitioning shapes into equal parts and naming those parts. They've also been limited to 2, 3, and 4 equal parts and using the words halves, thirds, and fourths. In third grade, it is crucial to take significant time to develop a full understanding of what a fraction is and how it is labeled in relation to the whole before they move into fourth, fifth, and sixth grades where they will be computing and solving real-world problems involving fractions.

In third grade, they are working with fractions with the denominators of 2, 3, 4, 6, and 8 only. They are representing fractions with models that include but are not limited to fraction bars, fraction circles, number lines, and pattern blocks. They are differentiating from unit fractions (1/b) and multiple units (a/b). Students should be able to use visual models and written and oral explanations to find equivalent fractions and explain why they are equivalent. Finally, they should compare fractions and justify their reasoning.

The key to developing this understanding is focusing on the visual models and providing time for students to justify all of their mathematical thinking. Our Fractions

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Progression (coming soon) will be very helpful in understanding the importance of taking the time to develop a full understanding, as opposed to giving “tips and tricks” for things like finding equivalent fractions and comparing them.

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Third Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Algebraic Thinking	**3.AT.1**: Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	High	Students will engage in 11 different problem types. Students will write an equation that matches the real world problem. Students will use strategies that include, but are not limited to number lines, arrow language, base-ten blocks, and partial sums to solve real world problems.
	3.AT.2: Solve real-world problems involving whole number multiplication and division within 100 in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	Medium	Student will use the correct terminology "groups of" "____ split into ___ equal groups" and "____ split into groups of ___." Students will distinguish the difference between two different division problem types. Students will use strategies that include but are not limited to drawings, base ten blocks, repeated addition, repeated subtraction, and properties of operations to solve real world problems. Students will solve problems using more than one strategy. Students will describe the strategies they use. Students will write an equation that matches the story problem. Students will identify the meaning of each number in the equation.
	3.AT.3: Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	Medium	Students will describe the question they need to Students will describe what they need to do in or Students will solve problems using more than on Students will describe the strategies they use. Students will explain what operation they used in
	3.AT.4: Interpret a multiplication equation as equal groups (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). Represent verbal statements of equal groups as multiplication equations.	Medium	Students will use the language "groups of" when describing multiplication problems. Students will distinguish the difference between 5 x 7 and 7 x 5. Students will depict multiplication problems using drawings, number lines, arrays, and repeated addition.
	3.AT.5: Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	Medium	Students will be able to finish number sentences such as: 6 x ___ = 48; ____ x 8 = 48; 6 x 8 = ___; 48 ÷ ___= 6; 48 ÷ ____ = 8; ___ ÷ 6 = 8; ___÷ 8 = 6. Students will descibe the relationship between multiplication and division using words, pictures, and models.

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	3.AT.6: Create, extend, and give an appropriate rule for number patterns using multiplication within 100.	Medium	Students will explain how doubling a factor doubles the product. Students will explain a factor can be decomposed and that partial products can be put back together. Students will see the relationship between factors (one can use their 2s to solve their 4s; their 3s to solve their 6s, etc.)
Data Analysis	**3.DA.1:** Create scaled picture graphs, scaled bar graphs, and frequency tables to represent a data set—including data collected through observations, surveys, and experiments—with several categories. Solve one- and two-step "how many more" and "how many less" problems regarding the data and make predictions based on the data.	Medium	Students will understand one square or picture on a graph does not have to equal 1. It can represent 5. Students will create a scaled picture graph with several catetories that represent data. Students will read and interpret scaled bar graphs in order to solve one- and two-step real world compare problems.
	3.DA.2: Generate measurement data by measuring lengths with rulers to the nearest quarter of an inch. Display the data by making a line plot, where the horizontal scale is marked off in appropriate units, such as whole numbers, halves, or quarters.	Medium	Students will create data by measuring lengths to the nearest quarter of an inch using a ruler. Students will display data involving lengths on a line plot. The line plot will be appropriately labeled with whole numbers, halves, and quarters.
	3.C.1: Add and subtract whole numbers fluently within 1000.	High	Students will add and subtract using strategies that include but are not limited to: manipulatives, pictures, number lines, arrow language, partial sums. Students will explain orally and through written language how they solve the problems. Students will explain how to add and subtract hundreds with hundreds, tens with tens, and ones with ones. Students will explain when and why it is necessary to compose and decompose hundreds and tens. Students will determine the most efficient way to arrive at the correct answer and explain why they feel it is the most efficient strategy.
	3.C.2: Represent the concept of multiplication of whole numbers with the following models: equal-sized groups, arrays, area models, and equal "jumps" on a number line. Understand the properties of 0 and 1 in multiplication.	High	Students will use the language "groups of" when describing multiplication problems. Students will depict multiplication problems using drawings, number lines, arrays, and repeated addition. Students desribe the properties of 0 and 1 when using these depictions.

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Computation	**3.C.3:** Represent the concept of division of whole numbers with the following models: partitioning, sharing, and an inverse of multiplication. Understand the properties of 0 and 1 in division.	High	Students will demonstrate sharing and partitioning in division (see 3.C.4) using strategies that include but are not limited to pictures, base ten, number lines, and arrays to depict division. Students will describe the properties of 0 and 1 when using these depictions and representations.
	3.C.4: Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).	Medium	Students will understand that division means "split into groups of" and "split into __ equal groups." (e.g. 56 ÷ 8 means 56 split into groups of 8 AND 56 split into 8 equal groups. Students will use written and oral language to differentiate between the two types of division.
	3.C.5: Multiply and divide within 100 using strategies, such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8), or properties of operations.	Medium	Students will multiply and divide using strategies such as pictures, base ten, repeated addition, repeated subtraction, number lines, and the inverse of mulitplication.
	3.C.6: Demonstrate fluency with multiplication facts and corresponding division facts of 0 to 10.	Medium	Students will use flexibility, accuracy, and efficiency in determining the best strategy for solving multiplication and division facts of 0 to 10.
Geometry	**3.G.1:** Identify and describe the following: cube, sphere, prism, pyramid, cone, and cylinder.	Low	Students will name the shape and traits of cubes, spheres, prisms, pyramids, cones, and cylinders. Students will identify these shapes in the world around them.
	3.G.2: Understand that shapes (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize and draw rhombuses, rectangles, and squares as examples of quadrilaterals. Recognize and draw examples of quadrilaterals that do not belong to any of these subcategories.	High	Students will use drawings, written explanations, and oral explanations to explain the similarities and differences of characteristics among the following quadrilaterals: rhombuses, rectangles, and squares. Students will create quadrilaterals that do not belong to the category of rhombus, rectangle, and square. Students will use written and oral language to explain why the quadrilateral does not belong to any of those categories.
	3.G.3: Identify, describe and draw points, lines and line segments using appropriate tools (e.g., ruler, straightedge, and technology), and use these terms when describing two-dimensional shapes.	High	Students will draw points, lines, and line segment Students will use written and oral explanations to Students will identify points, lines, and line segme
	3.G.4: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (1/2, 1/3, 1/4, 1/6, 1/8).	High	Students will partition any shape into equal areas and label each area as a unit fraction of the whole. Unit fractions will have the denominators of 2, 3, 4, 6, and 8.

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Measurement	**3.M.1:** Estimate and measure the mass of objects in grams (g) and kilograms (kg) and the volume of objects in quarts (qt), gallons (gal), and liters (l). Add, subtract, multiply, or divide to solve one-step real-world problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).	Medium	Students will distinguish the difference between grams and kilograms. Students will distinguish the difference between quarts, gallons, and liters. Students will solve two-step real world problems with mass and volume using the same unit of measurement. Students will represent real world problems involving mass using drawings.
	3.M.2: Choose and use appropriate units and tools to estimate and measure length, weight, and temperature. Estimate and measure length to a quarter-inch, weight in pounds, and temperature in degrees Celsius and Fahrenheit.	High	Students will recognize what they need to measure (length, weight, temperature) and determine what they need to use to do so Students will measure to a quarter inch Students will estimate length and weight
	3.M.3: Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real-world problems involving addition and subtraction of time intervals in minutes.	High	Students will tell time to the nearest minute Students will solve real world problems using time intervals in minutes Students will use various representations in adding and subtracting time including but not limited to pictures, number lines, and arrow language
	3.M.4: Find the value of any collection of coins and bills. Write amounts less than a dollar using the ¢ symbol and write larger amounts using the $ symbol in the form of dollars and cents (e.g., $4.59). Solve real-world problems to determine whether there is enough money to make a purchase.	High	Students will find the value of any combination of coins and bills. Students will represent money using appropriate symbols. Students will solve real world problems involving money, and, given the situation, determine if one has enough money to make a purchase. Students will use strategies that include but are not limited to manipulatives, drawings, number lines, and arrow language to determine whether or not they have enough money when solving real world problems.
	3.M.5: Find the area of a rectangle with whole-number side lengths by modeling with unit squares, and show that the area is the same as would be found by multiplying the side lengths. Identify and draw rectangles with the same perimeter and different areas or with the same area and different perimeters.	High	Students will understand what the area of a rectangle is. Students will model the area of a rectangle using objects and pictures. Students will see the connection between the construction of an array and finding the area of a rectangle. Students will understand how to find the area of a rectangle. Students will understand rectangles can have the same perimeter but different areas; they will represent their understanding using pictures, models, and written and oral explanation.

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	3.M.6: Multiply side lengths to find areas of rectangles with whole-number side lengths to solve real-world problems and other mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	Medium	Students will solve real world problems involving situations in which the area of something rectangular is found. Students will understand that the product of two whole numbers can be represented as rectangular.
	3.M.7: Find perimeters of polygons given the side lengths or by finding an unknown side length.	Medium	Students will understand what the perimeter of a polygon is. They will explain their understanding through pictures, words, models, and oral explanations. Students will find the missing length of a polygon when given the perimeter of a polygon.
Number Sense	**3.NS.1:** Read and write whole numbers up to 10,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 10,000.	High	Students will demonstrate their understanding of place value up to 10,000 through words, models, standard form, and expanded form. Students will rename numbers up to 10,000 in multiple ways.
	3.NS.2: Compare two whole numbers up to 10,000 using >, =, and < symbols.	Medium	Students will compare two whole numbers up to 10,000 using greater than, less than, and equal to symbols. Students will use models, pictures, and written and oral explanations to explain their mathematical thinking.
	3.NS.3: Understand a fraction, 1/b, as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction, a/b, as the quantity formed by a parts of size 1/b. [In grade 3, limit denominators of fractions to 2, 3, 4, 6, 8.]	High	Students will represent one part of the whole with correct frational notation. Fractions will be represented as 1/2; 1/3; 1/4; 1/6; and 1/8. Students will represent more than one part of the whole with denominators of 2, 3, 4, 6, and 8.
	3.NS.4: Represent a fraction, 1/b, on a number line by defining the interval from 0 to 1 as the whole, and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.	Medium	Students will understand there are numbers between 0 and 1. Students will partition the space between 0 and 1 into equal parts and label each part as 1/b (b represents the number of parts 0-1 has been partitioned into equally).
	3.NS.5: Represent a fraction, a/b, on a number line by marking off lengths 1/b from 0. Recognize that the resulting interval has size a/b, and that its endpoint locates the number a/b on the number line.	Medium	Students will understand there are numbers between 0 and 1. Students will partition the space between 0 and 1 into equal parts and label each part as 1/b (b represents the number of parts) AND more than one part is labeled as a/b (a represents how many parts).
	3.NS.6: Understand two fractions as equivalent (equal) if they are the same size, based on the same whole or the same point on a number line.	Medium	Students will use a number line to represent equivalent fractions.

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	3.NS.7: Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).	High	Students will use models that include but are not limited to number lines, fraction bars, fraction circles, and pattern blocks to represent equivalent fractions. Students will use written and oral explanations to justify why fractions are equivalent.
	3.NS.8: Compare two fractions with the same numerator or the same denominator by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	Medium	Students will using reasoning to compare fractions with the same numerator and different denominator. Students will compare fractions with a different numerator and same denominator. Students will represent their thinking with models and pictures. Students will justify their thinking with models, pictures, oral, and written language.
	3.NS.9: Use place value understanding to round 2- and 3-digit whole numbers to the nearest 10 or 100.	Medium	Students will round 2 and 3 digit whole numbers to the nearest 10 and 100. Students will use models that include but are not limited to number lines and place value knowledge to justify their reasoning for their final answer.

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Big Math Ideas

4th Grade

A mathematician's approach to the Indiana Academic Standards

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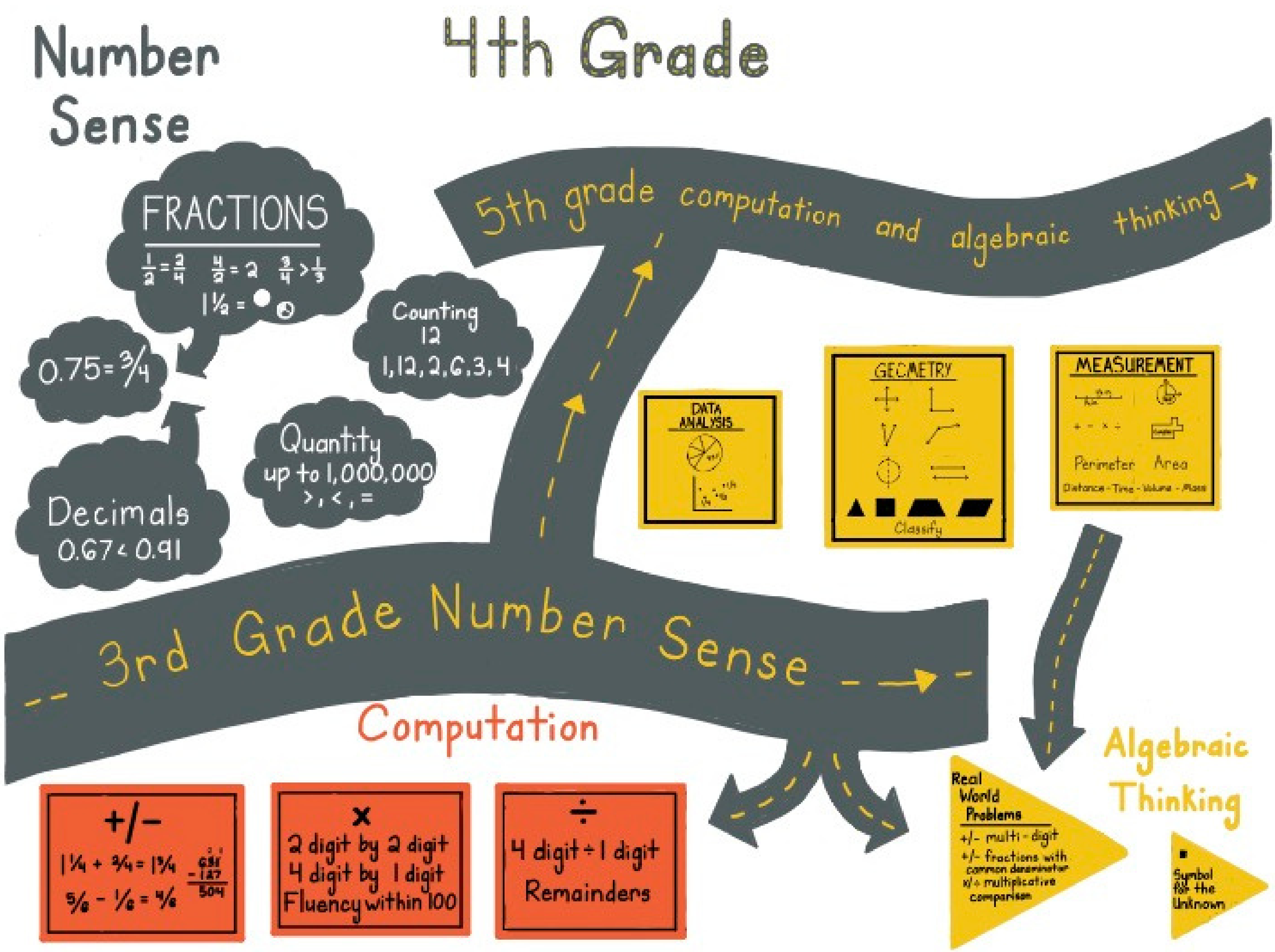
4th Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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4th Grade Big Math Ideas - Narrative

Children arrive in fourth grade equipped with the skills necessary to master adding and subtracting using the standard U.S. algorithm, multiplying and dividing larger numbers, continuing to grow their understanding of fractions as numbers, and developing an understanding of decimal notation. They will also solve many different kinds of real-world problems including problems with multiple steps.

The following big ideas are what teachers should spend the most time on in fourth grade. This doesn't mean that standards not mentioned aren't important. Rather, it's a guide to help educators see the importance of taking the time needed to develop these concepts before students move on to concepts in fifth grade.

Fractions and Decimals

In fourth grade, students need to continue building off the work they did in third grade as they work to deepen their understanding of **fractions** as numbers. As we explain below, this understanding is particularly important when students start to compute with fractions in fourth grade. Several concepts of fractions as numbers must be mastered in fourth grade. In teaching each fraction concepts, denominators should be limited to 2, 3, 4, 5, 6, 8, 10, 25, and 100.

1) Fractions can represent whole numbers. For example, 3/3 = 1 and 6/3 = 2.

2) Define a mixed number and convert mixed numbers into improper fractions *using objects and pictures.* Do not teach the traditional, procedural way of multiplying the whole number by the denominator and adding the numerator. Let students work to build understanding through finding the improper fraction using various models and describe why the improper fraction is the same as the mixed number. Skipping to the procedural steps will not help children as they continue to have to think flexibly about fractions as numbers.

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3) Explain why two fractions are equivalent when one multiplies the numerator and denominator by the same number. This is a procedural step, but why does it work? Students need to be able to explain this and justify their mathematical thinking.

4) Compare fractions with different numerators and denominators. Strategies to do this should include, but are not limited to, using visual fraction models such as fraction bars, fraction circles, number lines, and pattern blocks. Additionally, students should use their number sense and create a common denominator or numerator, or use a benchmark fraction. They must also be able to justify their mathematical thinking.

Be sure to check out our Computing Fractions and Decimals Progression (coming soon) to help you understand **why** spending this crucial time on computing fractions conceptually is necessary.

Prior to fourth grade, students have seen **decimals** in the form of money notation and are well versed in what this notation represents. They have also solved real-world problems involving money. Teachers should build off this knowledge when introducing the concept of tenths and hundredths. Use coins and dollars as a primary tool to understand 0.10 and how its relationship to 1.00. What form of money represents 1 tenth? What about 10 hundredths? Students can also use base ten blocks to develop their understanding of decimals. When the flat becomes equal to 1, the rod is now a tenth, and the unit a hundredth. With this, students can represent decimals in multiple ways, compare their values, and justify their thinking in doing so. In fifth grade, they will use these representations to compute decimals using all operations. Students are also asked to know the decimal equivalent to ½ and ¼. Think about how to teach this concept without telling them to divide 1 by 2. That is not developmentally appropriate for fourth graders (or fifth graders for that matter!).

Our Fractions and Decimals as Numbers Progression (coming soon) will help you see what students are coming to you with from grades K-3 and what they'll be doing in fifth grade. Also, check out the Computing Fractions and Decimals

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Progression (coming soon) to help you understand **why** spending this crucial time on fractions and decimals is necessary.

Addition and Subtraction

In fourth grade, students are **adding** and **subtracting** multi-digit whole numbers, fractions, and mixed numbers with common denominators. For whole numbers, it is time for them to use and describe the standard U.S. algorithm. When adding and subtracting fractions and mixed numbers, they should use strategies that include but are not limited to modeling with fraction bars, fraction circles, number lines, and pattern blocks. Students should also use what they know about decomposing numbers, finding equivalent fractions, and the relationship between addition and subtraction. It is crucial students are given the time to critically think through the process of adding and subtracting fractions and know how to represent this process in multiple ways. They should be able to explain and justify all of their mathematical thinking through written and oral language. Be sure to look at our Addition and Subtraction progressions (coming soon) to help you understand the importance of developing conceptual understanding in addition and subtraction prior to fourth grade and the work children have done in grades leading up to fourth.

Multiplication and Division

As a fourth grade teacher, it is very difficult to refrain from teaching the traditional U.S. algorithms for multiplication and division. However, this is NOT the expectation of fourth graders. The traditional U.S. **multiplication** algorithm is learned in fifth grade and the division algorithm in 6th grade. It is critical that 4th grade educators help students to hone the meanings of multiplication and division and do so with larger numbers.

First, students should be multiplying fluently within 100. Fluency does not mean "memorize" or quickly regurgitate facts. In fact, it's unnecessary for a child to be asked to solve 14 x 5 on demand. Rather, fluency means one must be flexible, accurate, and efficient in solving problems. Students need to **flexibly** think about ways in which they might multiply and divide. They must be able to **accurately**

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describe how they arrived at the correct answer, and they need to choose the strategy for arriving at the answer in the most **efficient** way. You can use simple problems like 14 x 5 to build area models and work on partial products. When they are given opportunities to do so, they will be able to think, "10 x 5 is 50, 4 x 5 is 20, 50 + 20 is 70." This is an example of a student thinking flexibly and developing fluency.

Students should also be multiplying up to four-digit numbers by one-digit and two-digit numbers by two-digits. They should do this using various strategies that include, but are not limited to, repeated addition, area model, and partial products. It is imperative children can use these algorithms and explain and justify all of their mathematical thinking. If they can do this, mastering the standard U.S. algorithm in fifth grade will be much easier! Be sure to look at our Multiplication Progression (coming soon) for more information on these concepts, as well as to understand the concepts children are coming to you with and why spending this time on these skills in fourth grade is so important as children move into fifth grade.

In **division**, students are continuing to build their understanding of the meaning of division (split into equal groups and split into groups of __). They will solve division problems with up to four-digit dividends and one-digit divisors using strategies that include but are not limited to repeated addition, repeated subtraction, base ten blocks, ratio tables, area model, and partial quotients. **DO NOT TEACH THEM the traditional U.S. algorithm**. They simply are not ready. When examining the knowledge students are coming with from third grade, educators will see students are not equipped with understanding why that algorithm works and their confusion caused by introducing the algorithm too early will cause them to shut down. Stick to the strategies above and leave the traditional U.S. algorithm to sixth grade teachers. They have time to learn. Do not rush these concepts! Students need a full understanding before they move into dividing more complicated numbers like fractions and decimals. Please see our Division Progression (coming soon) to help you see the process students need to go through to conceptually understand division of larger numbers!

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Real World Problem Solving

In fourth grade, children are responsible to solve real-world problems in the following ways:

- Adding multi-digit numbers
- Subtracting multi-digit numbers
- Multiplicative comparison (e.g., The building on the corner is 5 times the size as the building across the street. If the building on the corner is 85 stories tall, how tall is the building across the street?)
- Adding fractions with the same denominator
- Subtracting fractions with the same denominator
- Converting measurements given the larger unit to a smaller unit
- Situations involving money
- Situations involving time
- Situations involving area (including that of complex shapes)
- Situations involving perimeter

***Any of these problem types can be involved in two-step story problems.*

In order for students to truly understand real-world problem solving (and also work on the Process Standard 1 (PS 1): Make Sense of Problems and Persevere in Solving Them), they must be regularly given opportunities to solve such problems. A few things to consider:

1) Do not rely only on the textbook and when assigning real world problems. Often textbooks have the exact same problem type over and over again. Students should be solving problems where a number is missing from all parts of the equation:
    - 4 + 5 = ___
    - 4 + ___ = 9
    - ___ + 5 = 9
    - 9 - 5 = ___
    - 9 - ___ = 4
    - ___ - 5 = 4

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- 5 x 7 = ___
- 5 x ___ = 35
- ___ x 7 = 35
- 35 ÷ 5 = ___
- 35 ÷ ____ = 7
- ____ ÷ 5 = 7

***Any of the above examples can be substituted for a larger number or a fraction*

2) Students should be working through real-world problems at least 3 times a week. They should be provided time to work together, discuss strategies and outcomes, and share their thinking about how they arrived at the correct answer. Every time students solve real-world problems, there doesn't have to be a lesson before. Keep in mind, you don't have to teach a problem solving lesson each time you want your students to solve real-world problems. Instead, this practice should be consistent and ongoing so that students can develop strategies over time. Give them 10 - 15 minutes, 3 days a week to solve problems! Sometimes they'll be connected to a specific lesson on problem-solving, sometimes they won't!
3) Make sure they are using multiple strategies to solve their problems. All problem types can use models including but not limited to pictures, number lines, arrow language, and base ten representation. They should be able to explain all of their thinking and their entire process at arriving at their answers.
4) Stay away from "tips and tricks" and "key words." These do not help children with the PS 1 and impedes their ability to connect their work in fourth grade to grade levels beyond. Follow these steps:
    a) Read the problem.
    b) What's happening in the problem? What information do you have?
    c) What question do you need to answer?
    d) Look at the problem and determine the information you need to answer the question.
    e) Solve.
    f) Is your answer reasonable? If not, what do you need to rethink?

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These steps will work for all problem types, every time. Give children the opportunity to become critically thinking problem solvers. This will do nothing but help them as their learning in math continues.

5) Provide ample opportunities for students to write equations that represent problems they are solving. (e.g., Hinkle Fieldhouse holds 9,100 people when it is sold out. If they have sold 6,345 tickets, how many more tickets do they need to sell before they are sold out? 9,100 = 6,345 + ___). Students may use subtraction to *solve* the problem, but that is different from the equation that *represents* the problem. This notion is throughout all the standards and important to always include when giving story problem tasks.

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Fourth Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Algebraic Thinking	**4.AT.1:** Solve real-world problems involving addition and subtraction of multi-digit whole numbers (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	High	Students will engage in 11 different problem types. Students will write an equation that matches the real world problem. Students will use strategies that include, but are not limited to number lines, arrow language, base-ten blocks, and partial sums to solve real world problems.
	4.AT.2: Recognize and apply the relationships between addition and multiplication, between subtraction and division, and the inverse relationship between multiplication and division to solve real-world and other mathematical problems.	High	Students will understand that 3 groups of 5 can be written as 5 + 5 + 5 and 3 x 5. Students will understand that when dividing, one can subtract one group at a time to find out how many groups of _____ are in _____ (e.g. 52 ÷ 13 can be solved by doing 52 - 13 - 13 - 13 - 13). Students will understand that when they are solving a division problem, they can consider what they know about multiplication (e.g. when solving 52 ÷ 13, one can think 13 x ? - 52?) Students wll solve real world problems using the above strategies.
	4.AT.3: Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7, and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.	Medium	Students will explain how multiplication can compare quantities (e.g. Jason is 4 times as old as Beth. If Beth is 8, how old is Jason?) Students will justify their thinking through pictures, tape diagrams, and written and oral language.
	4.AT.4: Solve real-world problems with whole numbers involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), distinguishing multiplicative comparison from additive comparison. [In grade 4, division problems should not include a remainder.]	High	Students will solve real world problems using multiplication comparison (e.g. The building on the corner is 5 times the size as the building across the street. If the building on the corner is 85 stories tall, how tall is the building across the street?) Students will use models that include, but are not limited to pictures, tape diagrams, and number lines to solve the problems. Students will write equations that represent the problem. Students will use written and oral language to explain their mathematical thinking.

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	4.AT.5: Solve real-world problems involving addition and subtraction of fractions referring to the same whole and having common denominators (e.g., by using visual fraction models and equations to represent the problem).	High	Students will solve real world addition problems with common denominators and within the same whole (e.g. I have finished 2/4 of my book. If I want to finish 3/4 by the end of the day, how much more do I need to read?) Students will use models including but not limited to pictures, fraction bars, fraction circles, number lines, and pattern blocks to represent their mathematical thinking and solve the problem. Students will write and equation that matches the problem.
	4.AT.6: Understand that an equation, such as y = 3x + 5, is a rule to describe a relationship between two variables and can be used to find a second number when a first number is given. Generate a number pattern that follows a given rule.	Low	Students will understand an equation can have two variables in it. Students will solve two-variable equations when one variable is given. Students will identify patterns with equations.
Data Analysis	**4.DA.1:** Formulate questions that can be addressed with data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, and bar graphs.	High	Students will collect data and represent it using tables, frequency tables, line plots, and bar graphs. Students will write questions that can be answered from their data colletion. Students will make observations and interpret data as it is presented.
	4.DA.2: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using data displayed in line plots.	Medium	Students will design line plots based on measurement in fractions of 1/2, 1/4, and 1/8. Students will solve real world addition problems using the aformentioned types of line plots. Students will solve real world subtraction problems using the aformentioned types of line plots.
	4.DA.3: Interpret data displayed in a circle graph.	Medium	Students will understand how to read and int

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Computation	**4.C.1:** Add and subtract multi-digit whole numbers fluently using a standard algorithmic approach.	High	Students will connect base ten block representation to the standard US addition algorithm. Students will connect base ten block representation to the standard US subtraction algorithm. Students will solve addition problems using any algorithm including the standard US algorithm. Students will solve subtraction problems using any algorithm including the standard US algorithm. Students will explain through oral and written language how the standard US algorithm for addition and subtraction works. Students will attend to precision when explaining the standards US algorithm for addition and subtraction. (e.g. one does not "carry a one" rather, "When I add 9 + 7 in the ones column, I get 16. I am going to write down 6 in the ones column because 16 has 6 ones. Then, I will add the ten from 16 to the tens column. I will represent that ten with a 1 in the tens place.")
	4.C.2: Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Describe the strategy and explain the reasoning.	High	Students will multiply up to 4 digits by 1 digit numbers using strategies that include but are not limited to repeated addition, area model, and partial products. Students will mulitply 2 digit by 2 digit numbers using stategies that include but are not limited to repeated addition, area model, and partial products. Students will explain how the area model and partial products algorithm are related. Students will explain and justify all of their mathematical thinking through written and oral language. Students will explain how others solve up to four digit by 1 digit multiplciation problems and two digit by two digit multiplication problems.
	4.C.3: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning.	High	Students will solve division problems with up to four digit dividends and one digit divisors using strategies that include but are not limited to repeated addition, repeated subtraction, base ten blocks, ratio tables, area model, and partial quotients. Students will eplxain and justify their mathematical thinking through written and oral language.

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	4.C.4: Multiply fluently within 100.	Medium	Students will use flexibility, accuracy, and efficiency in determining the best strategy for solving multiplication problems with a product between 0 and 100.
	4.C.5: Add and subtract fractions with common denominators. Decompose a fraction into a sum of fractions with common denominators. Understand addition and subtraction of fractions as combining and separating parts referring to the same whole.	High	Students will add fractions with common denominators. Students will subtract fractions with common denominators. Students will decompose fractions with common denominators (e.g. 4/8 = 2/8 + 2/8). Students will model decomposing fractions with manipulatives that include but are not limited to fraction bars, fraction circles, and pattern blocks. Students will explain what they are doing when adding and subtracting fractions with common denominators using the same whole with pictures, manipulatives, and words.
	4.C.6: Add and subtract mixed numbers with common denominators (e.g. by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction).	High	Students will add and subtract mixed numbers with common denominators. Students will add and subtract mixed numbers with common denominators by modeling with manipulatives that include but are not limited to fraction bars, fraction circles and pattern blocks. Students will add and subtract mixed numbers with common denomintors using strategies that include but are not limited to finding equivalent fractions, using a number line, and understanding the relationship between addition and subtraction.
	4.C.7: Show how the order in which two numbers are multiplied (commutative property) and how numbers are grouped in multiplication (associative property) will not change the product. Use these properties to show that numbers can by multiplied in any order. Understand and use the distributive property.	Medium	Students will explain the commutative property of multiplication using numbers, pictures, and words. Students will explain the associative property of multiplication using numbers, pictures, and words. Students will explain the distributive property using numbers, pictures, and words.
	4.G.1: Identify, describe, and draw parallelograms, rhombuses, and trapezoids using appropriate tools (e.g., ruler, straightedge and technology).	Medium	Students will compare and contrast the characterstics of parallelograms, rhombuses, and trapezoids. Students will draw paralellograms, rhombuses, and trapezoids using tools such as rulers, straightedges, and other forms of technology.

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Geometry	**4.G.2:** Recognize and draw lines of symmetry in two-dimensional figures. Identify figures that have lines of symmetry.	Low	Students will identify and draw lines of symmetry in two-dimensional shapes. Students will explain through oral and written language why a shape is symmetrical.
	4.G.3: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.	Low	Students will identify angles in various geome
	4.G.4: Identify, describe, and draw rays, angles (right, acute, obtuse), and perpendicular and parallel lines using appropriate tools (e.g., ruler, straightedge and technology). Identify these in two-dimensional figures.	High	Students will describe, define, and draw rays, angles (right, acute, obtuse) and perpendicular and parallel lines using tools such as rulers, straightedges, and other technoglogy. Students will identify rays, angles, and perpendicular and paralell lines in two dimensional figures.
	4.G.5: Classify triangles and quadrilaterals based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles (right, acute, obtuse).	Medium	Students will distiguish the difference between triangles and quadrilaterals using attributes as described by parallel or perpendicular lines and/or right, acute, and obstuse angles. Student will use pictures, models, and written and oral language to justify their mathematical thinking.
	4.M.1: Measure length to the nearest quarter-inch, eighth-inch, and millimeter.	High	Students will measure the length of various objects to the nearest quarter-inch, eighth-inch, and millimeter.
	4.M.2: Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Express measurements in a larger unit in terms of a smaller unit within a single system of measurement. Record measurement equivalents in a two-column table.	High	Students will explain the relationship among units among one system of units, including km, m, cm; kg, lb, oz; l, ml; hr, min, sec. Students will express measurements in a larger unit in terms of a smaller unit (e.g. how many minutes in and hour; how many grams in kilograms, etc.) Students model measurement equivalents in a two column table.
	4.M.3: Use the four operations (addition, subtraction, multiplication and division) to solve real-world problems involving distances, intervals of time, volumes, masses of objects, and money. Include addition and subtraction problems involving simple fractions and problems that require expressing measurements given in a larger unit in terms of a smaller unit.	Medium	Students will solve real world addition, subtraction, multiplication, and division probems invovling measurements such as distance, time, volume, mass, and money. Students will solve the aforementioned with whole numbers and simple fractions. Students will find the larger unit in terms of the smaller unit in real world problems (e.g. I drove to my son's baseball game, and it took me an hour and fifteen minutes. How many minutes did my drive take?) Students will solve these real world problems using multiple strategies and justify their mathematical thinking through words, pictures, and various models.

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Measurement	**4.M.4:** Apply the area and perimeter formulas for rectangles to solve real-world problems and other mathematical problems. Recognize area as additive and find the area of complex shapes composed of rectangles by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts; apply this technique to solve real-world problems and other mathematical problems	High	Students will solve real world problems involving finding the area and perimeter of rectangles. Students will find the area of complex shapes by decomposing them into smaller rectangles. Students solve real world problems involving finding the area of complex shapes by decomposing them into smaller rectangles.
	4.M.5: Understand that an angle is measured with reference to a circle, with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. Understand an angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure other angles. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.	Low	Students will explain how an angle is represented in reference to a circle. (e.g. a 1 degree angle is 1/360 of a circle; a 90 degree angle is 90/360, or 1/4 of a circle; a straight angle (180 degrees) is 180/360, or 1/2 of a circle.
	4.M.6: Measure angles in whole-number degrees using appropriate tools. Sketch angles of specified measure	Medium	Students will measure angles using tools such as protractors and other shapes. Students will sketch angles when given a specific measurement.
	4.NS.1: Read and write whole numbers up to 1,000,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 1,000,000.	Low	Students will demonstrate their understanding of place value up to 1,000,000 through words, models, standard form, and expanded form. Students will rename numbers up to 1,000,000 in multiple ways.
	4.NS.2: Compare two whole numbers up to 1,000,000 using >, =, and < symbols.	Medium	Students will compare two whole numbers up to 1,000,000 using greater than, less than, and equal to symbols. Students will use models, pictures, and written and oral explanations to explain their mathematical thinking.

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Number Sense	**4.NS.3:** Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Name and write mixed numbers using objects or pictures. Name and write mixed numbers as improper fractions using objects or pictures.	High	Students will use pictures, words, and models to demonstrate whole numbers as fractions and fractions as whole numbers (e.g. 1 = 4/4; 10/2 = 5) Students will explain what a mixed number is using words, object, and pictures. Students will understand a mixed number with a numerator greater than a denominator is also called an improper fraction. Students will explain what an improper fraction is using words, models, and pictures.
	4.NS.4: Explain why a fraction, a/b, is equivalent to a fraction, (n × a)/(n × b), by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [In grade 4, limit denominators of fractions to 2, 3, 4, 5, 6, 8, 10, 25, 100.]	Medium	Students will demonstrate their understanding of equivalent fractions using words, pictures, and models. Manipulatives to demonstrate understanding include but are not limited to fraction bars, fraction circles, and pattern blocks. Denominators for finding equivalent fractions will only include 2, 3, 4, 5, 6, 8, 10, 25, and 100. Students will justify their mathematical thinking by explaning through written and oral language.
	4.NS.5: Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark, such as 0, 1/2, and 1). Recognize comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).	High	Students will compare fractions with different numerators and different denominators using strategies that include but are not limited to: pictures, models, words, tables, number lines, finding equivalent fractions, using a benchmark fraction. Students will justify their mathematical thinking using models and written and oral language.
	4.NS.6: Write tenths and hundredths in decimal and fraction notations. Use words, models, standard form and expanded form to represent decimal numbers to hundredths. Know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 = 0.50,	High	Students will demonstrate their understanding of decimal place value of tenths and hundredths using words, models, standard form, and expanded form. Models can include but are not limited to base ten blocks and money representation. Students will explain the decimal equivalent for halves and fourths using models and written and oral language.

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	4.NS.7: Compare two decimals to hundredths by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).	Medium	Students will compare decimals to hundredths using strategies that include but are not limited to: pictures, models, words, number lines, and money representations. Students will justify their mathematical thinking using written and oral language.
	4.NS.8: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.	Medium	Students will use words, pictures, and numbers to differentiate between "factor" and "multiple." Students will find all factor pairs of whole numbers from 0-100. Students will recognize patterns in finding factors pairs (e.g. if a number is a factor of 4, it is also a factor of 8). Students will explain why these patterns and relationships exist using words, numbers, and pictures. Students will justify why numbers are factors of larger numbers using words, numbers, and pictures.
	4.NS.9: Use place value understanding to round multi-digit whole numbers to any given place value.	Medium	Students will round multi-digit whole numbers to any given place value. Students will use models that include but are not limited to number lines and place value knowledge to justify their reasoning for their final answer.

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Big Math Ideas

5th Grade

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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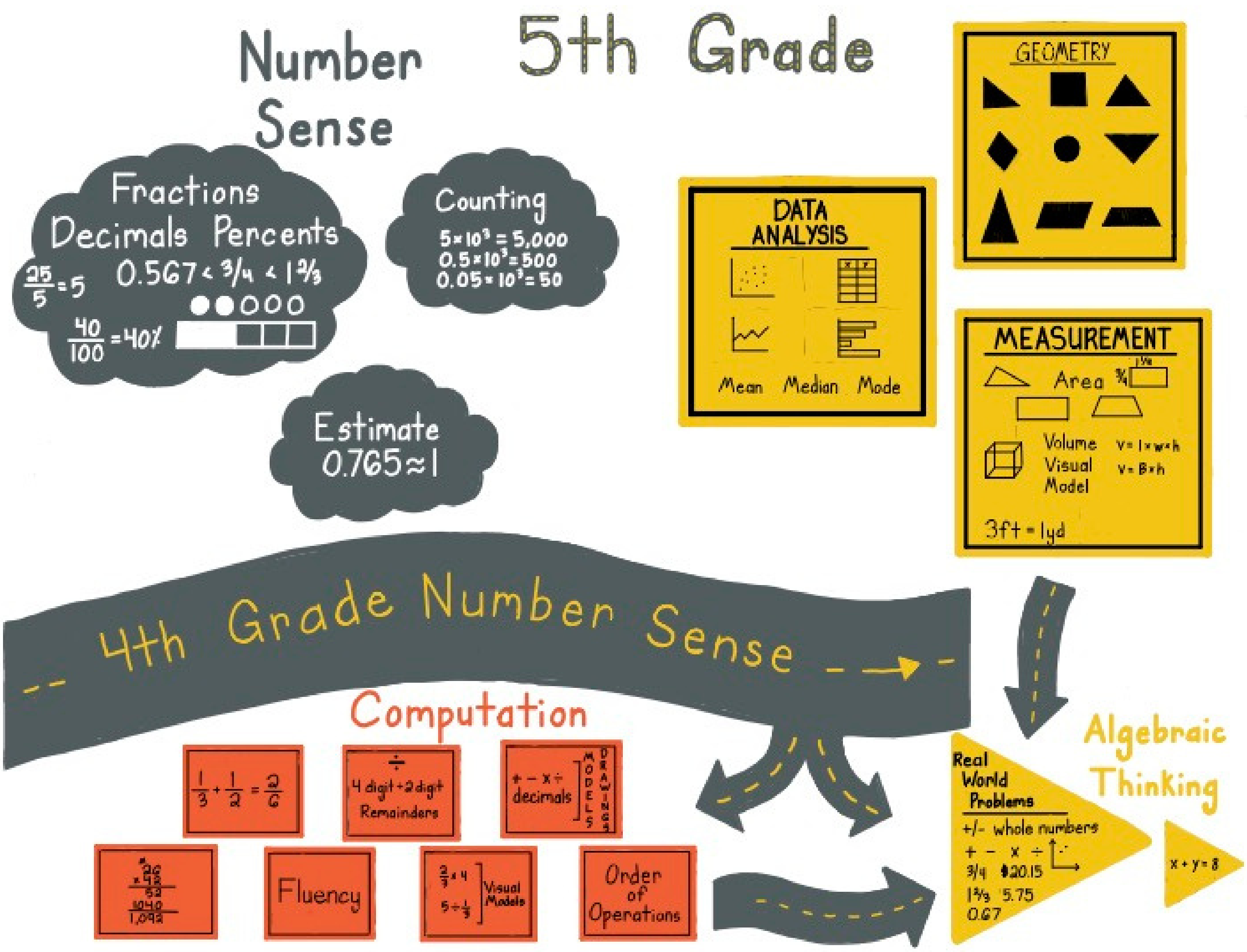
5th Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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5th Grade Big Math Ideas - Narrative

Children arrive in fifth grade equipped with the skills necessary to master the standard U.S. multiplication algorithm, divide larger numbers using multiple strategies, add, subtract, multiply and divide fractions and decimals. All of these skills will connect to solving real-world problems, as there is a great deal of real-world problem solving happening in fifth grade. Students will also make connections among many different geometry and measurement concepts and hone their knowledge of fractions and decimals.

The following big ideas are what educators should spend the most time on in fifth grade. This doesn't mean that standards not mentioned aren't important. Rather, it's a guide to help illustrate the importance of taking the time needed to develop these concepts before students move on to deeper concepts in sixth grade.

Computation

In fourth grade, students added fractions and mixed numbers with common denominators. In fifth grade, they will move into adding fractions and mixed numbers with unlike denominators. However, it's important that students aren't taught only one way to do this (traditionally students are taught to find a common denominator and add). Rather, they should use their number sense about fractions and finding equivalent fractions to **add** and **subtract**. It is also a misconception that answers must be simplified. Students need to recognize answers can come in multiple forms (e.g. 3 ½ is the same as 7/2 is the same as 3 2/4).

Students will also add and subtract decimals to to hundredths. All numbers in these computation problems can go up to hundredths. As well, students need to think the same ways they did to add and subtract whole numbers, leaning on whole-number computation strategies to solve decimal computation problems. Students should understand how to add hundredths to hundredths, tenths to tenths, ones to ones, etc. If composing a number and a place value to the left is needed,

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students should know to do so. Give them a chance to make sense of this process. They are NOT just "carrying a one." Something very specific is happening with the numbers, and students must be able to explain this.

Be sure to check out our Fractions and Decimals as Numbers Progression (coming soon) and our Computing Fractions and Decimals Document (coming soon) to understand the processes you want your students to go through as they are working these concepts!

Multiplication and Division

Students in fifth grade are multiplying and dividing many different kinds of numbers.

First of all, fifth grade is the time to teach the U.S. standard multiplication algorithm with whole numbers. This should be done in ways that support conceptual understanding of the algorithm. Students should be able to connect the previously learned partial products algorithm to the standard algorithm and explain what is happening as they go through the steps of the algorithm. When working through the steps, numbers should be called what they are (e.g. when you multiply 6 x 7 in the ones place, you are not "carrying" a 4 to the tens, you are adding 4 tens to the tens place). Be sure students can explicitly explain all of these steps and WHY each step is taking place. Conceptual understanding of the algorithm is not the same as being able to regurgitate the steps. Our Multiplication Progression (coming soon) will help you visualize this process!

Students are also asked to multiply fractions by a fraction or by a whole number using visual models. Let the kids show their mathematical thinking using tools like fraction bars, fraction circles, number lines, and pattern blocks. Start with multiplying fractions by whole numbers. Make sure they are interpreting the problem exactly as it is written (e.g. 5 x ¾ is the same as 5 groups of ¾). Allow students to build the problem and depict the answer in multiple ways. How does this representation transfer to thinking about ¾ of a group of 5? Give them time to build, think, and

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discuss to develop understanding. Do not just tell them to multiply across the top and bottom. They'll figure that out as they are developing a conceptual understanding of multiplying fractions.

Students also need to be multiplying decimals. Numbers with digits to the hundredths column can be used in both factors. Give them a chance to do some estimating before they begin thinking about solving answers (e.g. 3.25 x 6.78 is about the same as 3 x 7. Therefore, the answer should be somewhere around 21.) This will help them as they make sense of what to do with the decimal. Don't just teach the standard algorithm. Again, this is a sixth grade standard. Leave it to the sixth grade teachers, but equip the kids with the why so the standard algorithms make sense. Please look through our Computing Fractions and Decimals Progression (coming soon) to understand the full process children are going through with fractions and decimals!

In division, students are continuing to build their understanding of the meaning of division (split into equal groups and split into groups of __). Now, they are doing so with up to four digit dividends by two digit divisors using strategies that include but are not limited to repeated addition, repeated subtraction, base ten blocks, ratio tables, area model, and partial quotients. **DO NOT TEACH THEM the traditional U.S. algorithm**. They simply are not ready! Let them make sense of division with larger numbers and leave the teaching of the traditional algorithm to 6th grade educators. When looking carefully at the knowledge fifth graders are coming with, you will see that many are not yet equipped with understanding why the algorithm works and their confusion with the algorithm will shut them down. They have time to learn this algorithm in 6th grade. Do not rush these concepts! They need a full understanding of division before they move into dividing more complicated numbers like fractions and decimals (which they start in fifth grade!)

Students will also be dividing fractions. However, READ CAREFULLY. They are only dividing a whole number by a unit fraction (fraction with a one in the numerator) and a unit fraction by a whole number. Use the same thinking as is discussed above with multiplication. Use the language the kids are equipped with to

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talk through what is happening in the division of fractions (e.g., 5 ÷ ½ is the same as 5 split into groups of ½). Students should model problems to develop an understanding of what they are dividing. DO NOT teach them to “keep, change, flip.” There is a ***mathematical reason*** why this works. Your job as a fifth grade teacher is to help students understand the foundations of dividing fractions. Have them look for patterns and think about the math they are doing. Sixth grade teachers teach the standard algorithm and why it works. Please look through our Computing Fractions and Decimals Progression (coming soon) to understand the full process children are going through with dividing whole numbers, fractions, and decimals!

Real-World Problem Solving

In fifth grade, children are responsible to solve real-world problems in the following ways:

- Multiplying multi-digit whole numbers
- Dividing multi-digit whole numbers and interpret the remainder
- Adding fractions with unlike denominators
- Subtracting fractions with unlike denominators
- Multiplying fractions including mixed numbers
- Dividing unit fractions
- Adding decimals to hundredths including situations involving money
- Subtracting decimals to hundredths including situations involving money
- Multiplying decimals to hundredths including situations involving money
- Dividing decimals to hundredths including situations involving money (divisor should remain a whole number)
- Situations involving measurement conversions
- Situations involving find the area of triangles, parallelograms, and trapezoids
- Situations involving finding the perimeter of triangles, parallelograms, and trapezoids
- Situations involving the volume of a rectangular prism

***Any of these problem types can be involved in two-step story problems.*

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In order for students to truly understand real-world problem solving (and also work on the Process Standard 1 (PS 1): Make Sense of Problems and Persevere in Solving Them), they must be regularly given opportunities to solve real-world problems. A few things to consider:

1) Do not rely only on the textbook when assigning real-world problems. Often textbooks have the exact same problem type over and over again. Students should be solving problems where number is missing from all parts of the equation:
    - 4 + 5 = ___
    - 4 + ___ = 9
    - ___ + 5 = 9
    - 9 - 5 = ___
    - 9 - ___ = 4
    - ___ - 5 = 4
    - 5 x 7 = ___
    - 5 x ___ = 35
    - ___ x 7 = 35
    - 35 ÷ 5 = ___
    - 35 ÷ ____ = 7
    - ____ ÷ 5 = 7

***Any of the above examples can be substituted for a larger number, fraction, or decimal.*

2) Students should be working through real-world problems at least 3 times a week. They should be provided time to work together, discuss strategies and outcomes, and share their thinking about how they arrived at the correct answer. Keep in mind, you don't have to teach a problem solving lesson each time you want your students to solve real-world problems. Instead, this practice should be consistent and ongoing so students can develop strategies over time." Give them 10 - 15 minutes, 3 days a week to solve problems!
3) Make sure they are using multiple strategies to solve their problems. All problem types can use models including but not limited to pictures, number

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lines, arrow language, and base ten representation. They should be able to explain all of their thinking and their entire process at arriving at their answers.

4) Stay away from "tips and tricks" and "key words." These do not help children with the PS 1 and impedes their ability to connect their work in fifth grade to grade levels beyond. Follow these steps:
    a) Read the problem.
    b) What's happening in the problem? What information do you have?
    c) What question do you need to answer?
    d) Look at the problem and determine the information you need to answer the question.
    e) Solve.
    f) Is your answer reasonable? If not, what do you need to rethink?

   These steps will work for all problem types, every time. Give children the opportunity to become critically thinking problem solvers. This will do nothing but help them as their learning in math continues.

5) Provide ample opportunities for students to write equations that represent problems they are solving. (e.g., Hinkle Fieldhouse holds 9,100 people when it is sold out. If they have sold 6,345 tickets, how many more tickets do they need to sell before they are sold out? 9,100 = 6,345 + ___). Students may use subtraction to *solve* the problem, but that is different from the equation that *represents* the problem. This notion is throughout the standards and important to always include when giving story problem tasks.

Geometry and Measurement

The geometry and measurement standards in fifth grade are plentiful. There are ample opportunities for students to make connections among many different geometrical concepts and apply them to real-world situations (see above).

Students are asked to classify shapes and understand the overlaps among various polygons and their relationships (e.g., all squares are quadrilaterals, but all quadrilaterals are not squares). They also need to attend to precision as they are

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describing the characteristics of shapes and the types of angles and number of sides they include.

Students are asked to **understand** the formulas for finding the area of triangles, parallelograms, and trapezoids. However, there is no need to teach the formulas with no context. Provide opportunities for them to make connections to area formulas they learned in already know (e.g., the area of a rectangle was learned in third grade and applied to finding the area of complex shapes in fourth) and to build and manipulate shapes to uncover what formulas might work to find the area of the polygon (e.g. ½ b x h is related to a rectangle in what way?). Teaching formulas and having students plug in numbers isn't helping them understand the math behind the formulas, so the formula won't stick in their mind. Give them ample opportunity to think and apply. Kids can do this work!

Fractions, Decimals, and Percents

Prior to fifth grade, students have worked very hard to understand fractions and decimals as numbers by representing them in multiple ways. They know they can decompose fractions and decimals just as they can whole numbers, and they can think about what they know about money to represent numbers like 20/100 and 0.20.

With this, students are asked to plot and compare fractions, mixed numbers, and decimals on a number line. They can use this skill to round decimals to any given place value. Students also need to differentiate between fractions as part of a whole (⅖ of a book), fractions as part of a set (⅖ of the kids in line have brown eyes), and fractions as representation of division (10/2 = 5).

Fifth grade is also the first time students will hear about percents. During this time, they are simply asked to understand percents from the perspective of a part of 100 and to do so through pictures, diagrams, and real-world examples. Take some time to check out our Fractions and Decimals as Numbers Progression (coming

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soon) to see what children are coming to you with from prior grades and what the expectations are in sixth grade.

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Fifth Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
	5.AT.1: Solve real-world problems involving multiplication and division of whole numbers (e.g. by using equations to represent the problem). In division problems that involve a remainder, explain how the remainder affects the solution to the problem.	High	Students will solve real world multiplication and division problems. Division problems will include measurement division and equal sharing problems. Students will write an equation that matches the real world problem. Students will use multiple strategies to arrive at the correct answer. Students will explain their mathematical thinking through written and oral language. When the real world problem involves division with remainders, students will represent the remainder appropriately and explain how the remainder impacts the answer to the problem.
	5.AT.2: Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models and equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess whether the answer is reasonable.	High	Students will solve real world addition and subtraction problems involving fractions with like and unlike denominators. Students will solve for the unknown in all locations. Students will write an equation that matches the real world problem. Students will solve problems using fraction models that include but are not limited to fraction bars, fraction circles, number lines, benchmark fractions, and pattern blocks. Students will apply their knowledge of the value of fractions to justify whether or not answers are reasonable. Students will explain their thinking through written and oral language.
	5.AT.3: Solve real-world problems involving multiplication of fractions, including mixed numbers (e.g., by using visual fraction models and equations to represent the problem).	Medium	Students will solve real world multiplication problems inovlving fractions including mixed numbers. Students will write an equation that matches the real world problem. Students will solve problems using fraction models that include but are not limited to fraction bars, fraction circles, number lines, benchmark fractions, and pattern blocks. Students will explain their thinking through written and oral language.

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Algebraic Thinking	**5.AT.4:** Solve real-world problems involving division of unit fractions by non-zero whole numbers, and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem).	Medium	Students will solve real world division problems involving dividing unit fractions (where 1 is the numerator) by whole numbers. Students will solve real world division problems involving dividing whole numbers by unit fractions (where 1 is the numerator). Students will write equations that match the real world problem. Students will solve problems using fraction models that include but are not limited to fraction bars, fraction circles, number lines, benchmark fractions, and pattern blocks. Students will explain their thinking through written and oral language.
	5.AT.5: Solve real-world problems involving addition, subtraction, multiplication, and division with decimals to hundredths, including problems that involve money in decimal notation (e.g. by using equations to represent the problem).	High	Students will solve real world addition, subtraction, multiplication, and division problems involving decimals to the hundredths. Students will have experience with 14 different problem types. Students wil solve real world addition, subtraction, multiplication, and division problems involving money in decimal notation. All numbers in all equations will go up to the hundredths place, except when using the divisor which should only be a whole number. Students will write equations that represent the real world problem. Students will use strategies that include but are not limited to estimation, number lines, base ten blocks, pictures, arrow language, partial sums, partial products, and partial quotients. Students will explain their thinking through written and oral language.
	5.AT.6: Graph points with whole number coordinates on a coordinate plane. Explain how the coordinates relate the point as the distance from the origin on each axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	Medium	Students will recognize the coordinate plane and define the origin. Students will differentiate between the x-axis and the y-axis. (the x-axis you travel left and right and the y-axis you travel up and down). Students will know a coordinate pair is (x,y) and plot it on a coordinate plane. Students will explain the relationship between ordered pairs and their location on the coordinate plane (e.g. a point moves to the right as the number in the x coordinate increases and a point moves higher when the y coordinate number increases. 2,10 is above 2,5 because they have the same x coordinate and the y coordinate is a greater number)

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	5.AT.7: Represent real-world problems and equations by graphing ordered pairs in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Low	Students will use coordinate pairs in the first quadrant of the coordinate plane to represent real world problems.
	5.AT.8: Define and use up to two variables to write linear expressions that arise from real-world problems, and evaluate them for given values.	High	When given a real world problem, students will write and use an equation with two variables. (e.g. Juan is x years old. How old will he be in 5 years? [x + 5] His brother is n years younger, how much older is Juan than his brother? [x - n]).
	5.C.1: Multiply multi-digit whole numbers fluently using a standard algorithmic approach.	High	Students will connect area model and partial sums to the standard US algorithm for multiplication. Students will explain through oral and written language how any algorithm, including the standard US algorithm for multiplication works. Students will attend to precision when explaining the standard US algorithm for multiplication (e.g. one does not "carry a one"; rather, if they multiply 9 x 2 in the ones column, they add the ten from 18 to the tens column).
	5.C.2: Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning used.	High	Students will divide up to four-digit numbers by up to two-digit numbers using strategies that include but are not limited to repeated addition, repeated subtraction, base ten blocks, ration tables, area model, and partial quotients. Students will eplxain and justify their mathematical thinking through written and oral language."
	5.C.4: Add and subtract fractions with unlike denominators, including mixed numbers.	High	Students will add and subtract fractions with unlike denominators. Students will use a variety of strategies that include but are not limited to finding an equivalent fraction resulting in a common denominator and finding a least common denominator. Numbers should include mixed numbers and improper fractions. Students will understand answers can be represented in multiple ways. Answers to not always have to be simplified and improper fractions do not always need to be made mixed numbers. Students will find the missing numerator or denominator in an equation.

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Computation	**5.C.5:** Use visual fraction models and numbers to multiply a fraction by a fraction or a whole number.	Medium	Students will multiply a fraction by a whole number and a whole number by a fraction using strategies that include but are not limited to use of fraction bars, fraction circles, pattern blocks, and number lines. Students will articulate the meaning of multiplying a fraction (e.g. 4 x 2/3 means four groups of 2/3 and 2/3 x 4 means 2/3 a group of 4) Students will justify and explain their mathematical thinking using written and oral language.
	5.C.6: Explain why multiplying a positive number by a fraction greater than 1 results in a product greater than the given number. Explain why multiplying a positive number by a fraction less than 1 results in a product smaller than the given number. Relate the principle of fraction equivalence, $a/b = (n \times a)/(n \times b)$, to the effect of multiplying a/b by 1.	Medium	Students will explain the size of a product compared to the factors. (e.g. how does the product of 2/3 x 3 compare to 3? What do you notice when you multiply 1/2 x 4, 1/ 2 x 2, 1/2 x 1, and 1/2 x 1/2?) Students will explain why a product changes when any number is multiplied by a factor greater than 1or a factor less than one (e.g. what happens when you multiply 4 x 8? What happens when you multiply 4 x 1/8?) Students will explain that when multiplying the numerator and denominator by the same number, one is multiplying by 1 (e.g. 4/5 x 1 is the same as 4/5 x 6/6)
	5.C.7: Use visual fraction models and numbers to divide a unit fraction by a non-zero whole number and to divide a whole number by a unit fraction.	Medium	Students will divide a unit fraction (a fraction with a numerator of 1) by a whole number and a whole number by a unit fraction using strategies that include but are not limited to use of fraction bars, fraction circles, pattern blocks, pictures, and number lines. Students will explain their mathematical thinking using written and oral language.
	5.C.8: Add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value or the properties of operations. Describe the strategy and explain the reasoning.	Medium	Students will add, subtract, multiply, and divide decimals to hundredths using strategies that include but are not limited to partial sums, partial products, partial quotients, pictures, base ten blocks, number lines, and arrow language. All combinations of whole numbers and decimals should be computed (e.g. whole numbers in all parts of the equation up to decimals to hundredths in all parts of the equation).
	5.C.9: Evaluate expressions with parentheses or brackets involving whole numbers using the commutative properties of addition and multiplication, associative properties of addition and multiplication, and distributive property.	Low	Students will solve problems enacting the rules of order of operations. Students will understand why order of operations is a mathematical convention.

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Data Analysis	**5.DS.1:** Formulate questions that can be addressed with data and make predictions about the data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, bar graphs, and line graphs. Recognize the differences in representing categorical and numerical data.	Low	Students will design questions that gather data and make predictions about outcomes. Students will create ways to gather data such as observations, surveys and experiements. Students will collect data and present it using tables, graphs, and other representations. Students will differentiate between categorical data (e.g., favorite subject in school) and numerical data (e.g., number of jumps with a jump rope in one minute).
	5.DS.2: Understand and use measures of center (mean and median) and frequency (mode) to describe a data set.	High	Student will explain the difference between fractions that represent a part of a whole (e.g., 2/5 of the book was read), a part of a set (e.g. 2/5 of the kids in line have brown eyes), and division of whole numbers (e.g. 10/2 = 5).
Geometry	**5.G.1:** Identify, describe, and draw triangles (right, acute, obtuse) and circles using appropriate tools (e.g., ruler or straightedge, compass and technology). Understand the relationship between radius and diameter.	High	Students will identify right, acute, and obtuse triangles. Students will compare and contrast the characteristics of right, acute, and obtuse triangles. Students will draw right, acute, and obstuse triangles using appropriate tools. Student will understand the difference between a radius and diameter of a circle.
	5.G.2: Identify and classify polygons including quadrilaterals, pentagons, hexagons, and triangles (equilateral, isosceles, scalene, right, acute and obtuse) based on angle measures and sides. Classify polygons in a hierarchy based on properties.	High	Students will classify 3-, 4-, 5-, and 6-sided shapes. Students will classify shapes using characteristics of angle measures and sides. Students will generate categories (e.g., all square are nested within the 'quadrilaterals, ' but not all quadrilaterals are squares).
	5.M.1: Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step real-world problems.	High	Students will convert measurements within various systems (e.g. distance, volume, mass, etc.) from larger unit to smaller unit and smaller unit to larger unit. Students will apply conversions to real world problems.

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Measurement	**5.M.2:** Find the area of a rectangle with fractional side lengths by modeling with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	High	Students will find the area of a rectangle when sides are represented by fractions. Students will understand square units can be represented by fractions. Students will model with unit squares to represent the area of a rectangle. Students will understand they can multiply a fraction by a fraction to find the area of a rectangle.
	5.M.3: Develop and use formulas for the area of triangles, parallelograms and trapezoids. Solve real-world and other mathematical problems that involve perimeter and area of triangles, parallelograms and trapezoids, using appropriate units for measures.	High	Students will develop the formula for the area of a triangle by utlizing knowledge of the area of a rectangle. Students will develop the formula for the area of a parallelogram by utilizing the area of a rectangle and triangle. Students will develop the formula for the area of a trapezoid by utilizing knowledge of the area of a rectangle, triangle, and pallelogram. Students will solve real world problems involving area of triangles, parallelograms, and trapezoids.
	5.M.4: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths or multiplying the height by the area of the base.	High	Students will apply what they know about finding the area of a rectangle to develop the formula for finding the volume of a rectangular prism. Students will model the volume of a rectangular prism using unit cubes. Students will explain how V= l xl wl xl h is the same as V = B x h
	5.M.5: Apply the formulas V = l × w × h and V = B × h for right rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve real-world problems and other mathematical problems.	High	Students will use the forumulas for volume to solve real
	5.M.6: Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems and other mathematical problems.	Low	Students will find the volume of irregular shapes composed of non-overlapping rectangular prisms. Students will apply finding the volume of irregular shapes to real world situations.
	5.C.3: Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	Low	Students will estimate the size of products by comparing the size of its factors (e.g. 5 x 3 is less than 52 x 3). Students will estimate the size of factor based on the product.

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Number Sense	**5.NS.1**: Use a number line to compare and order fractions, mixed numbers, and decimals to thousandths. Write the results using >, =, and < symbols.	High	Students will plot fractions, mixed numbers, and decimals on a number line to compare the value of numbers. Students will use words and pictures to justify their mathematical thinking on placement of numbers on the number line.
	5.NS.2: Explain different interpretations of fractions, including: as parts of a whole, parts of a set, and division of whole numbers by whole numbers.	High	Student will explain the difference between fractions represent a part of a whole (e.g., 2/5 of the book was read), a part of a set (e.g. 2/5 of the kids in line have brown eyes), and division of whole numbers (e.g. 10/2 = 5).
	5.NS.3: Recognize the relationship that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right, and inversely, a digit in one place represents 1/10 of what it represents in the place to its left.	High	Students will understand that moving a digit one place to the left multiplies its value by 10. Students will understand that moving a digit one place to the right divides its value by 10.
	5.NS.4: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	Medium	Students will understand powers of 10. Students will understand that appending a zero on the far right end of a number increases by a power of 10 (e.g. 100 -> 1,000). Students will understand why when multiplying a decimal by 10, the decimal moves one place to the right (e.g. 2.36 x 10 = 23.6) Students will understand why when dividing a decimal by 10, the decimal moves one place to the left (e.g. 23.6 ÷ 10 = 2.36) Students will explain and justfy the movement of a decimal when multiplied by a multiple of ten (or power of 10) using words, pictures, and diagrams.
	5.NS.5: Use place value understanding to round decimal numbers up to thousandths to any given place value.	High	Students will round decimals up to thousandths to any given place value. Students will use models that include but are not limited to number lines and place value knowledge to justify their reasoning for their final answer.
	5.NS.6: Understand, interpret, and model percents as part of a hundred (e.g. by using pictures, diagrams, and other visual	High	Students will explain what a percent is as it relates to a number out of 100. Students will use picture, diagrams, and other tools to model the meaning of percentages. Students will use written and oral language to explain percents out of 100.

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Big Math Ideas

6th Grade

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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with

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May 2022

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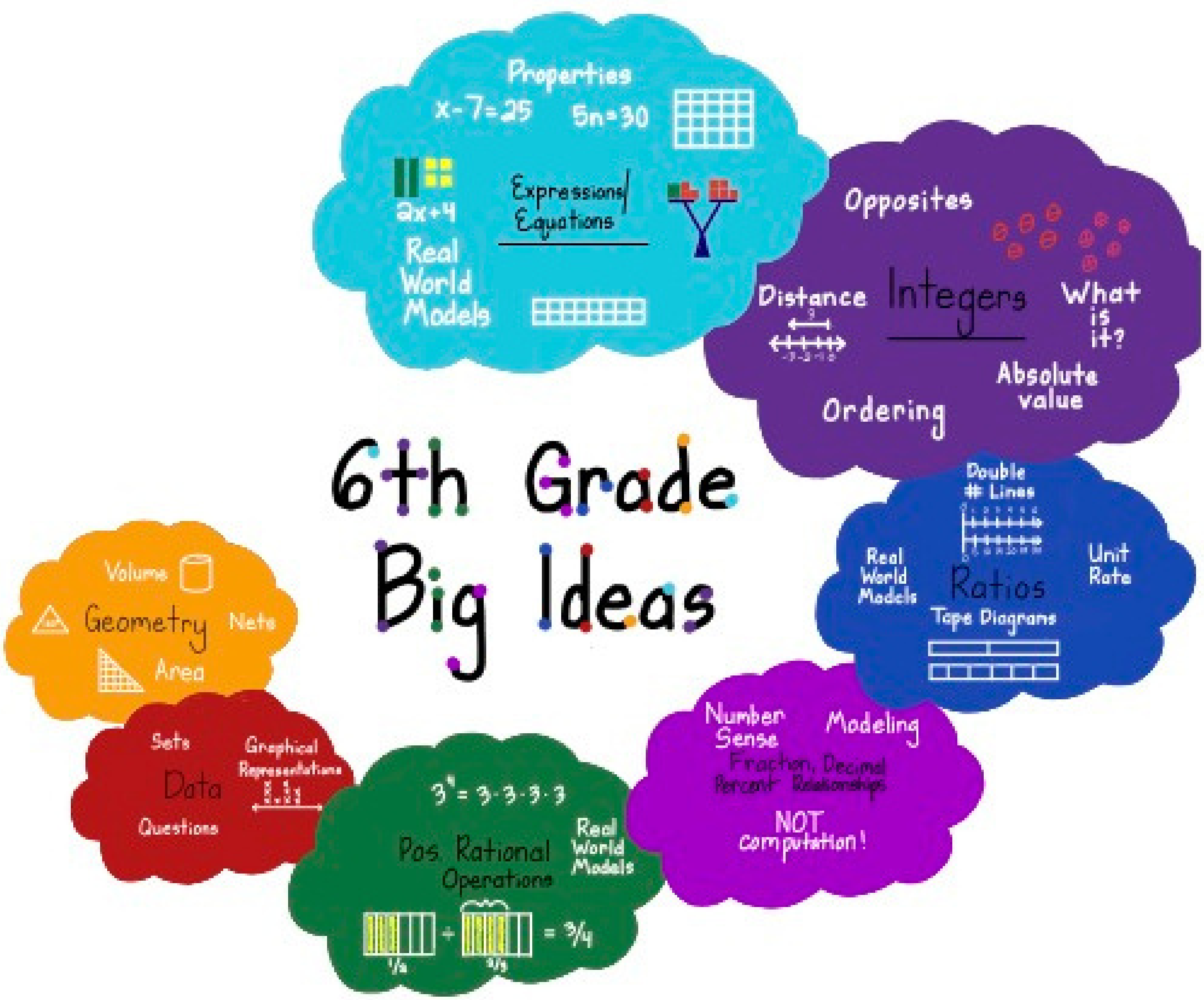
6th Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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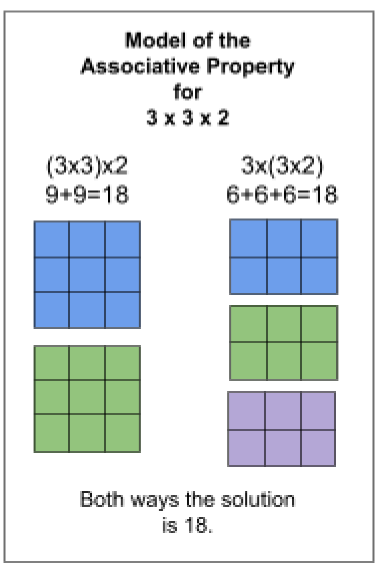
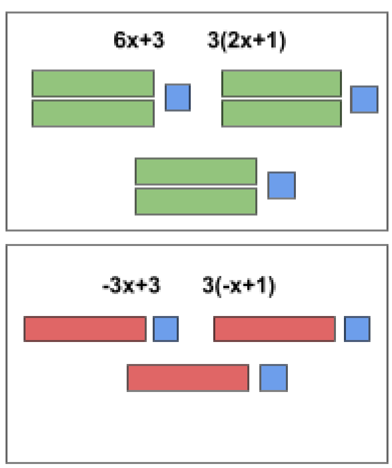
6th Grade Big Math Ideas - Narrative

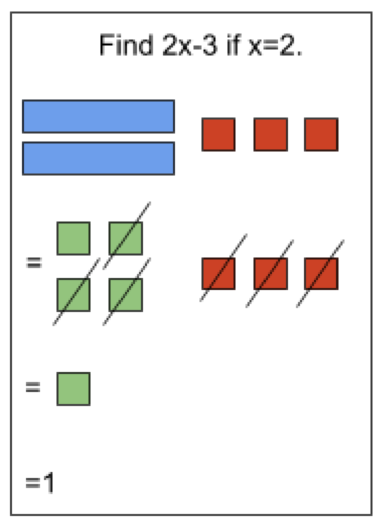
The following narrative will guide you through the Big Math Ideas diagram above. A common theme to look for throughout the 6th grade curriculum is **modeling**, both with manipulatives and with real-world situations emphasizing evaluating expressions.

Expressions and Equations

We will start with the larger, more important topic of expressions. Expressions should be modeled heavily at this level (see below). Expressions should be used to apply properties and show how math "works;" hence there are rules governing what we do in math. Students learn what "like terms" are, and we introduce students to variables with an equal sign. Students have worked with symbols for the unknown and have been introduced to the term 'variable, but this is where their understanding is formalized. It is important this change is taught conceptually so students begin their equation work with a solid understanding of what an equation is. Algebra tiles are a great way to accomplish this and should be used throughout middle school.







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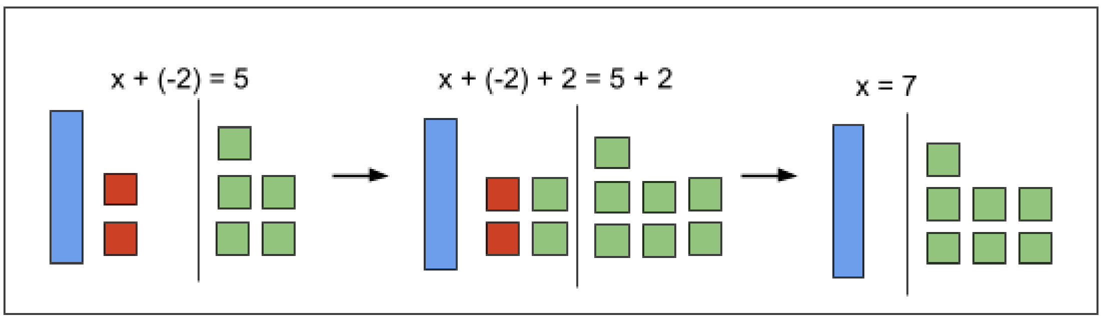
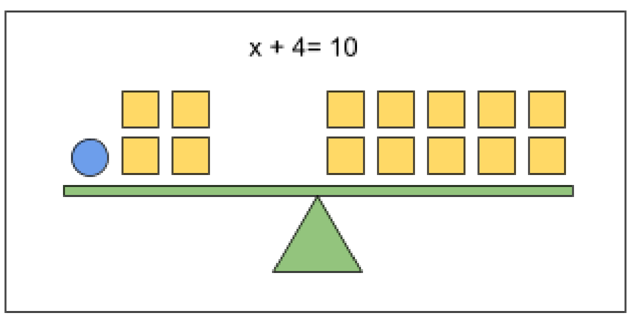
Once students have a firm grasp on expressions, simplifying them through manipulatives and algebraic thinking, we introduce equality and 1-step equations. Students should begin the study of equations with whole numbers by using visual models such as scales, algebra tiles, counters, and drawings. Students will then add tables and graphs to their solution methods. Inverse operations should be added last and be introduced by using visual models. Equations with fractions and decimals can be added into the mix and real-world models will be a major part every step along the way. These should be modeled heavily using tables, graphs, inverse operations, scales, algebra tiles (as noted above), other manipulatives and real-world examples.





The idea of inequalities is also introduced in 6th grade, using number lines to illustrate and build conceptual understanding of what it means to have multiple solutions. The Big Math Idea students are developing with inequalities and the introduction of a variable is that there are multiple solutions to the inequality; the Big Math Idea is not the algebraic manipulation by using inverse operations. Focus your efforts on using a number line or other visuals to solve these inequalities in 6th grade.

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Integers

Introduction to integers is also a Big Math Idea in 6th grade. 6th graders need to learn what a negative number is in relation to the left side of zero on the number line; however, NO OPERATIONS with integers are taught in 6th grade. Models, models, models. They need a good, conceptual foundation before they hit the operations in 7th grade. Students should be exploring opposites, ordering, absolute value and distance using number lines.

Number Sense

Another important idea 6th graders learn is the relationship between fractions, decimals, and percents in the domain of Number Sense. This is achieved by heavily modeling what numbers look like across the three representations. At this point, teachers should be modeling equivalence only, and **not focusing on the computational conversion from one form to another.** Modeling could and should be done using double number lines and tape diagrams. Students need a good, working understanding of common fractions with denominators of 2, 3, 4, 5, 8, and 10 along with their decimal and percent equivalents. Under the Number Sense strand, students should be able to move fluently between different representations of common fractions, for example, knowing that ¾ is the same as .75 and 75%. They should be able to do this *without* a calculator. As well, they should not be given fractions like 5/7 and asked for the decimal equivalent. Students may use a calculator to do some conversion between representations when they compare rational numbers for 6.NS.3, but this Big Idea is not about conversion unless they can do it without a calculator, hence the focus on only the common, small denominators listed.

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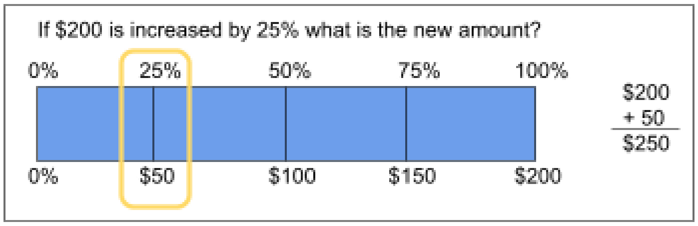

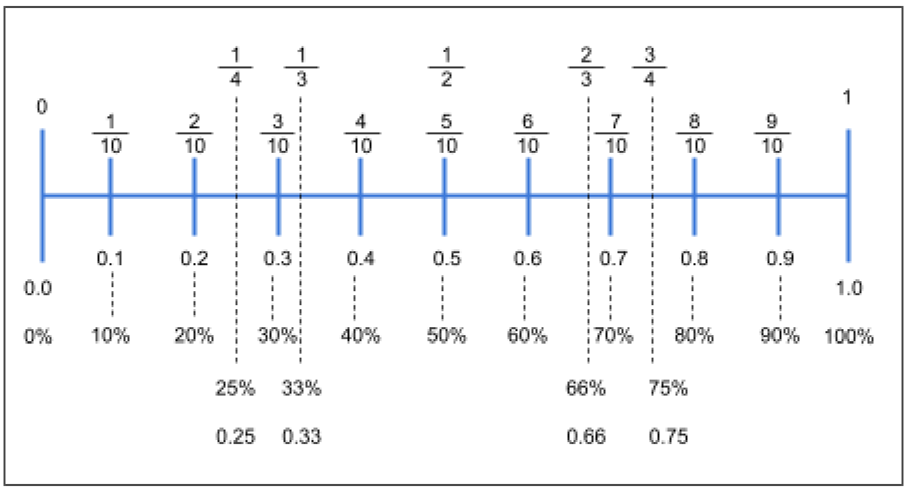

Computation

Under the domain of Computation, formalized division is a **high priority standard** and should be taught with real-world models. Computing with positive fractions and decimals is also important and, again, should be done using real-world applications, NOT drill and kill with the standard algorithm. That practice does little to strengthen any conceptual understanding of fractions and decimals, and the learning is not transferable to new and unique situations. The Big Math Idea with both division and computing with fractions and decimals is the understanding of how those concepts fit into the real world. These are mastery standards; therefore students need to create real-world models, and solve new and unique situations

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they have never seen before while applying division. In particular, division of fractions should be done with visual fraction models (see examples from ScaffoldedMath.com and this video by Math with Mr. J) and used to solve real-world problems. Mechanical manipulations such as Keep Change Flip! do not increase conceptual understanding and should be avoided. (**See these articles: #1, #2**) Positive whole number exponents are also introduced in 6th grade.

Though not tested as a high priority standard on ILEARN, ratios are a Big Math Idea when it comes to understanding middle school mathematics. It is of special note to make sure students understand that ratios are *not fractions*. Visual models such as tape diagrams, double number lines, and drawings, along with tables and graphs, are especially important here and, as always, real-world models need to be a major focus of the ratio work. Unit rate and proportional relationships are also introduced here.

Coordinate Graphing is another lesser topic in 6th grade moving students to all four quadrants instead of just quadrant 1. Learning to plot points and looking at the distance between points on vertical or horizontal lines of the graph are introduced.

Data

Although data is identified as a **medium priority standard** for ILEARN, its importance for middle school mathematics cannot be understated. This work is foundational for the work students will do later in high school and college, when taking classes like Finite Math, Quantitative Analysis, Statistics, and Discrete Math. Data in 6th grade includes data collection, graphical representations of the data, and interpretation of the data as it appears in a variety of representations with the use of technology. The unit on data should be taught at the beginning of the 2nd semester so students have time to master the concepts. Often teachers wait until May to get to data "if they have time," and too often, students leave for summer without mastery of important content in this strand.

Geometry is a lesser 6th grade topic, but again, this does not indicate that it should be ignored or skipped. It does mean it does not need the same attention,

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rigor, and testing as a topic like expressions requires. 6th graders should have exposure and strong DOK 1 and 2 level understanding of area of complex shapes, the sum of the interior angles of a triangle, volume of right rectangular prisms, and nets and how they are used to understand surface area of right rectangular prisms.

Modeling using tape diagrams, double number lines, drawings, tables, graphs, and real-world situations in 6th grade is of the utmost importance to give students a solid, conceptual understanding of the topics .

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Sixth Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
	6.AF.1: Evaluate expressions for specific values of their variables, including expressions with whole-number exponents and those that arise from formulas used in real-world problems.	High	Understand and solve new and unique real-world problems that include expressions evaluated for a specific numeric value that could include whole number exponents.
	6.AF.2: Apply the properties of operations (e.g., identity, inverse, commutative, associative, distributive properties) to create equivalent linear expressions and to justify whether two linear expressions are equivalent when the two expressions name the same number regardless of which value is substituted into them.	Medium	Apply the properties of operations to create equivalent expressions and justify why they are equivalent.
	6.AF.3: Define and use multiple variables when writing expressions to represent real-world and other mathematical problems, and evaluate them for given values.	High	Write experessions to represent new and unique real-world problems using multiple variables, such as distance (d) times time (t), and evaluate them for given values.
	6.AF.4: Understand that solving an equation or inequality is the process of answering the following question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	Medium	Demonstrate understanding that the solution to an equation or inequality is a number that makes the math sentence true. Test numbers from a set to find out if it is a solution to an equation or inequality.
	6.AF.5: Solve equations of the form $x + p = q$, $x - p = q$, $px = q$, and $x/p = q$ fluently for cases in which p, q and x are all nonnegative rational numbers. Represent real-world problems using equations of these forms and solve such problems.	High	Represent and solve real-world problems with one-step equations fluently and flexibly (with tables, algebra tiles, graphs, scales, manipulatives, inverse operations) with positive rational numbers.

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Algebra and Functions	**6.AF.6:** Write an inequality of the form x > c, x ≥ c, x < c, or x ≤ c, where c is a rational number, to represent a constraint or condition in a real-world or other mathematical problem. Recognize inequalities have infinitely many solutions and represent solutions on a number line diagram.	Medium	Understand that an inequality has infinite solutions. Write an inequality to represent a real-world situation, and illustrate it on a number line.
	6.AF.7: Understand that signs of numbers in ordered pairs indicate the quadrant containing the point; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. Graph points with rational number coordinates on a coordinate plane.	Medium	Graph points accurately on a coordinate plane. Identify the four quadrants by the signs of the ordered pairs.
	6.AF.8: Solve real-world and other mathematical problems by graphing points with rational number coordinates on a coordinate plane. Include the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Medium	Solve real-world problems by plotting points on the coordinate plane. Find the distance between two points with the same first coordinate or the same second coordinate.
	6.AF.9: Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane.	Medium	Create ratio tables with whole number measurements. Find missing values in a ratio table. Plot the ordered pairs from a ratio table on the coordinate plane.
	6.AF.10: Use variables to represent two quantities in a proportional relationship in a real-world problem; write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	Medium	Write an equation in two variables to represent a real-world situation showing the relationship between the independent and dependent variables. Tell what the independent and dependent variables are given a graph, table, or equation.
	6.C.1: Divide multi-digit whole numbers fluently using a standard algorithmic approach.	High	Divide whole numbers fluently in the context of real-world situations in order to make the skill transferable.

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Computation	**6.C.2:** Compute with positive fractions and positive decimals fluently using a standard algorithmic approach.	High	Compute with positive fractions and decimals fluently in the context of real-world situations in order to make the skill transferable.
	6.C.3: Solve real-world problems with positive fractions and decimals by using one or two operations.	Medium	Solve real-world problems with positive fractions and decimals using one or two operations
	6.C.4: Compute quotients of positive fractions and solve real-world problems involving division of fractions by fractions. Use a visual fraction model and/or equation to represent these calculations.	Medium	Students will understand division of fractions conceptually and illustrate with visual models like tape diagrams. Compute quotients of positive fractions.
	6.C.5: Evaluate positive rational numbers with whole number exponents.	Low	Students can evaluate exponents with positive rational bases and whole number exponents
	6.C.6: Apply the order of operations and properties of operations (identity, inverse, commutative properties of addition and multiplication, associative properties of addition and multiplication, and distributive property) to evaluate numerical expressions with nonnegative rational numbers, including those using grouping symbols, such as parentheses, and involving whole number exponents. Justify each step in the process.	Medium	Students can apply the order of operations and explain what they are doing at each step using properties of operations. Students can evaluate expressions with grouping symbols and whole number exponents.
Data Analysis and Statistics	**6.DS.1:** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for the variability in the answers. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	Medium	Students can recognize the relationship between a statistical question and the data collected to answer that question. Students can describe a set of data by its center, spread, and overall shape.
	6.DS.2: Select, create, and interpret graphical representations of numerical data, including line plots, histograms, and box plots.	Medium	Students will understand that there are different ways to represent numerical data. Students can interpret data from a line plot, histogram, and box plot in the context of a real-world model.

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	6.DS.3: Formulate statistical questions; collect and organize the data (e.g., using technology); display and interpret the data with graphical representations (e.g., using technology).	Medium	Students can formulate a question, collect data, and display that data (using technology) to answer the question.
	6.DS.4: Summarize numerical data sets in relation to their context in multiple ways, such as: report the number of observations; describe the nature of the attribute under investigation, including how it was measured and its units of measurement; determine quantitative measures of center (mean and/or median) and spread (range and interquartile range), as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered; and relate the choice of measures of center and spread to the shape of the data distribution and the context in which the data were gathered.	Medium	Students can summarize numerical data sets in multiple ways such as: ~the number of observations ~how the attribute was measured ~quantitative measures of center and spread ~describe overall patterns as well as any striking deviations ~relate the choice of measures to the shape of the data distribution
	6.GM.1: Convert between measurement systems (English to metric and metric to English) given conversion factors, and use these conversions in solving real-world problems.	Low	Understand that in the context of solving some real-world problems, we sometimes need to convert between measurement systems. Convert between measurement systems given conversion factors.
	6.GM.2: Know that the sum of the interior angles of any triangle is 180° and that the sum of the interior angles of any quadrilateral is 360°. Use this information to solve real-world and mathematical problems.	Medium	Students can explain why the sum of the angles of a triangle is 180. Students can explain why the sum of the angles of a quadrilateral is 360. Students can use the sum of the interior angles of a triangle or quadrilateral to solve real-world problems.
	6.GM.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate; apply these techniques to solve real-world and other mathematical problems.	Medium	Students can use vertical and horizontal lines of figures on the coordinate plane to solve real-world problems.

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Geometry and Measurement	**6.GM.4:** Find the area of complex shapes composed of polygons by composing or decomposing into simple shapes; apply this technique to solve real-world and other mathematical problems.	Medium	Students can compose or decompose complex shapes into simple shapes. Find the area of complex shapes in real-world contexts.
	6.GM.5: Find the volume of a right rectangular prism with fractional edge lengths using unit cubes of the appropriate unit fraction edge lengths (e.g., using technology or concrete materials), and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = lwh and V = Bh to find volumes of right rectangular prisms with fractional edge lengths to solve real-world and other mathematical problems.	Medium	Students can illustrate volume of a right rectangular prism with technology and derive the formula from the illustration. Use the formulas for volume to solve real-world problems.
	6.GM.6: Construct right rectangular prisms from nets and use the nets to compute the surface area of prisms; apply this technique to solve real-world and other mathematical problems.	Medium	Students can compute the surface area of right rectangular prisms from nets. Students can use nets to solve real-world problems.
	6.NS.1: Understand that positive and negative numbers are used to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). Use positive and negative numbers to represent and compare quantities in real-world contexts, explaining the meaning of 0 in each situation.	High	Students will understand the concept of negative numbers, opposites, and where they are used in real-world contexts. Students should be able to illustrate negative numbers in a real-world situation using manipulatives or drawings.
	6.NS.2: Understand the integer number system. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself (e.g., −(−3) = 3), and that 0 is its own opposite.	Medium	Students can explain that opposites are the same distance from zero on opposite sides of zero. Students can show that the opposite of the opposite of a number is the number itself.

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Number Sense	**6.NS.3**: Compare and order rational numbers and plot them on a number line. Write, interpret, and explain statements of order for rational numbers in real-world contexts.	Medium	Students can compare rational numbers in real-world contexts. Students can order rational numbers on a number line.
	6.NS.4: Understand that the absolute value of a number is the distance from zero on a number line. Find the absolute value of real numbers and know that the distance between two numbers on the number line is the absolute value of their difference. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	Medium	Understand that absolute value is a number's distance from zero on a number line. Students can find the absolute value of a real number.
	6.NS.5: Know commonly used fractions (halves, thirds, fourths, fifths, eighths, tenths) and their decimal and percent equivalents. Convert between any two representations (fractions, decimals, percents) of positive rational numbers without the use of a calculator.	Medium	Students know and can show common fractions and their decimal and percent equivalents using tape diagrams or number lines. Students can convert between these representations WITHOUT the use of a calculator.
	6.NS.6: Identify and explain prime and composite numbers.	Low	Students can define prime and composite numbers and identify commonly used prime numbers.
	6.NS.7: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers from 1 to 100, with a common factor as a multiple of a sum of two whole numbers with no common factor.	Low	Find the GCF of two numbers less than 100 using *any method*. Find the LCM of two numbers 12 or less using *any method*.
	6.NS.8: Interpret, model, and use ratios to show the relative sizes of two quantities. Describe how a ratio shows the relationship between two quantities. Use the following notations: a/b, a to b, a:b.	Medium	Demostrate conceptual understanding of what a ratio is by explaining the relationship of the two quantities. Model a ratio in a variety of ways, including words, pictures, and numbers.
	6.NS.9: Understand the concept of a unit rate and use terms related to rate in the context of a ratio relationship.	Low	Demonstrate understanding of a unit rate by using the correct terms in the context of the relationship described.

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	6.NS.10: Use reasoning involving rates and ratios to model real-world and other mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).	Medium	Model ratios in real-world situations in a variety of ways, including ratio tables, tape diagrams, double number lines, or equations.

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Big Math Ideas

7th Grade

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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with

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May 2022

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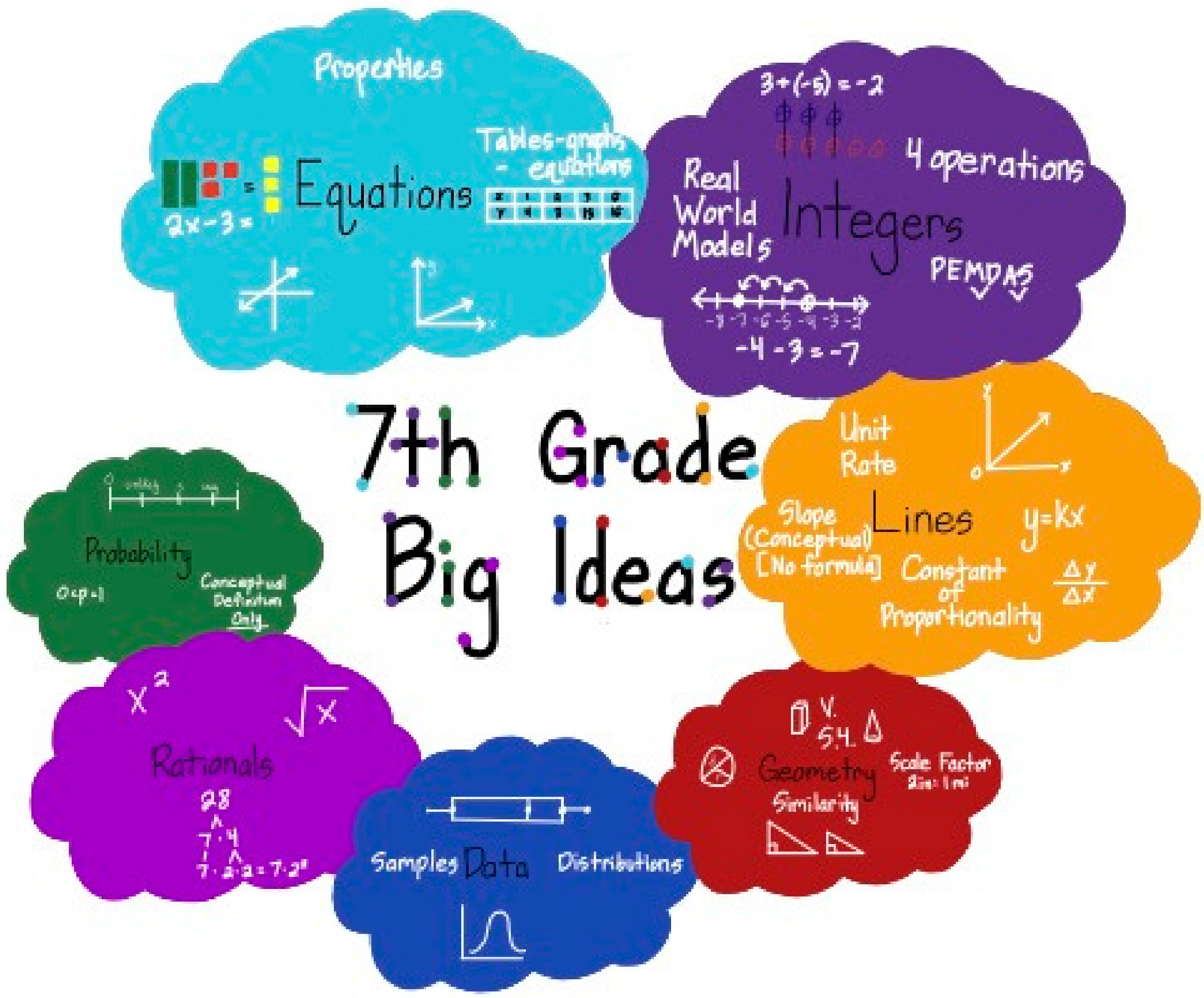
7th Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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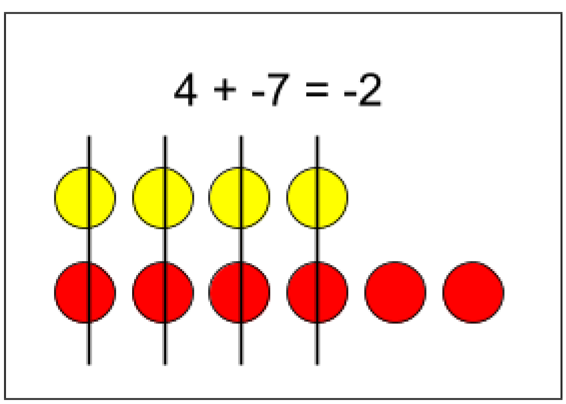
7th Grade Big Math Ideas - Narrative

The following narrative will guide you through the Big Ideas diagram above. The theme is "model, model, model."

Integers

Operations with integers, though listed as a medium priority for ILEARN because of how it is tested, is most definitely a **high priority** for middle school mathematics success. It is important to note integer operations must be taught conceptually at the onset and then move to the procedural algorithms **when** students are ready. Students need to understand, for example, why two negatives result in a positive when multiplied together. It is important for students to understand this concept rather than remember a mnemonic device or rhyme which helps them memorize the rule. (**See these articles: #1, #2**) This will ensure a more complete understanding for students along with a higher retention and transfer rate later. Teachers should use models such as number lines, integer chips, temperature and other real-world examples in their lessons. There should be a focus on real-world models that tie integer operations to the students' prior knowledge. It cannot be emphasized enough that until students master a thorough **understanding** of integers and their operations, they should not be shown the standard algorithms for the operations.



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Solving Equations: 2 Step Equations

In 7th grade, we emphasize algebra as a big idea with solving equations as we move into two-step equations and inequalities. Modeling with algebra tiles, scales, tables and graphs FIRST is critical (see examples below). Students must understand, conceptually, how equations work before simply learning an algorithm. The purpose of solving equations is not to solve them using inverse operations; rather, it is to understand the answer to the equation or inequality is the number(s) that makes the equation or inequality true. Students should, most importantly, be able to relate this to a real-world situation. Worksheet after worksheet where students are solving multiple equations does not develop this Big Math Idea. The goal of working with equations in 7th grade is learning to solve them with tables and graphs from real-world models and understanding what the solution represents in the context of the situation. Yes, students who understand the inverse operations method can engage with that method, but it should not be the method we lead with. Let students continue to use the method they understand at their level of learning and scaffold them to inverse operations. Giving students more than one way to look at, understand, and solve a problem ensures access and equity for more students. Students don't all learn at the same rate, at the same time, or at the same level. Multiple entry points to a problem gives more students a chance to see themselves as mathematicians and changes the negative self-perceptions that students often have about their mathematical ability.

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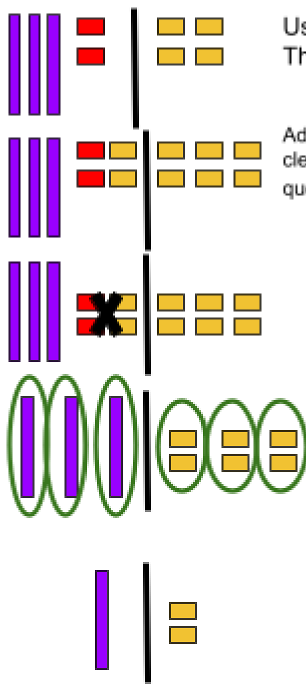
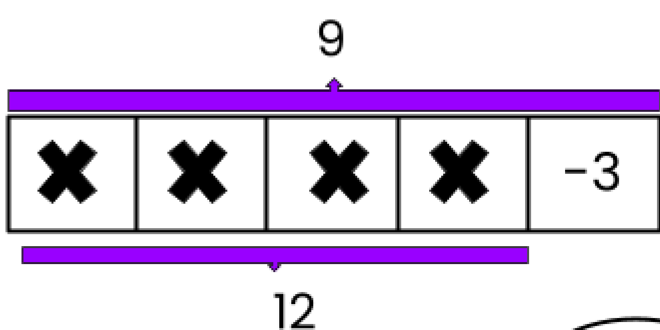


$3x - 2 = 4$	Use tiles to represent the equation. The purple represents x.
$3x - 2 + 2 = 4 + 2$	Add 2 to each side. Be sure students are clear why they are making this step. Ask questions to guide the process.
$3x = 6$	Simplify
$\frac{3x}{3} = \frac{6}{3}$	Divide each side into groups of 3. Remember to continue asking questions about they "why" as you go through this process!
$X = 2$	

$4x - 3 = 9$



$X = 3$

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Proportional Relationships

Learning about ratios in 6th grade leads to a comparison of ratios in 7th grade. This comparison, or relationship, between ratios is a proportion. Students should be first introduced to proportional reasoning through real-world modeling. Students study unit rate in 6th grade, while in 7th grade they begin to see that this unit rate can become a constant rate across the real-world model on a graph. For example, consider a proportional relationship between time and distance in a situation such as, "How long does it take a car to go from home to the store?" If the car travels at a constant unit rate, that becomes a constant rate of change and the graph is linear. If the car starts at home, we could consider that starting point the origin of the graph so the constant rate of change becomes the constant of proportionality. Students in 7th grade should only graph lines in the form y = mx, where there is no y-intercept so the rate of change is constant and becomes the constant of proportionality.

All of this conceptual understanding must come through real-world modeling, not teaching rules and definitions. The study of linear equations and graphing linear equations should include the use of tables; just as we used them to solve the equations, we will now use them to graph the equations. Students should NOT be taught the term 'slope' in 7th grade or any definition of slope. All references to the constant rate of change that happens in a linear relationship should be the "rate of change". This phrase is much more universal when talking about the relationship between the vertical and horizontal change of a line. What we typically think of as proportion problems, the missing value problems where we cross multiply and divide, should not be a focus in this unit. Students should be taught to use proportional reasoning, not immediately jump to the shortcut of cross multiplication, as it adds no mathematical value to their understanding of the concept. Students can use a ratio table, a graph or an equation to solve a proportional problem.

We measured the height of the flowers in inches in the front and backyard over the period of 12 days.

Time	**2 days**	**4 days**	**6 days**	**8 days**	**10 days**	**12 days**

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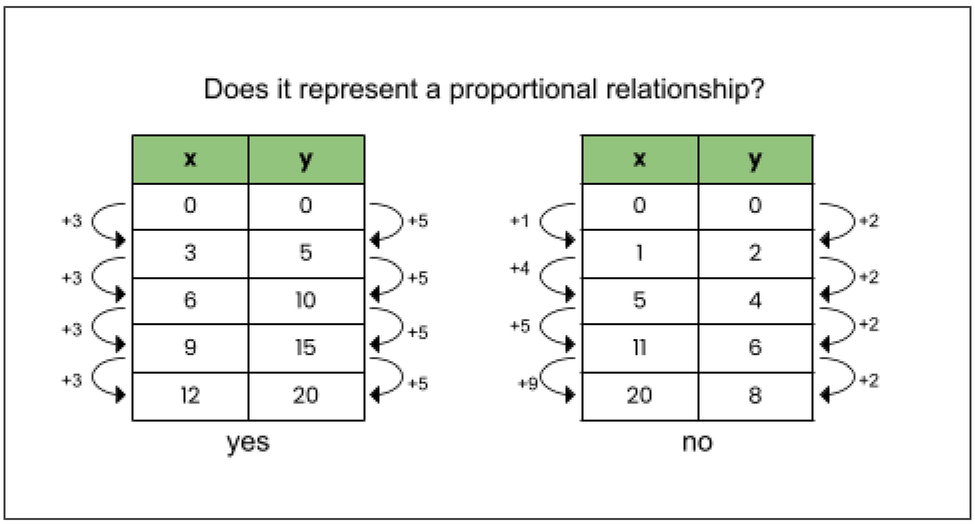
Height of front yard flowers	1/2	1	1 1/4	1 1/2	2	2 1/2
Height of backyard flowers	1 1/4	2 1/2	3 1/8	3 1/4	5	5 1/4

Are the height of the flowers in the backyard always proportional to the height of the flowers in the front yard?



Geometry

Scale factor is the most important idea in geometry in 7th grade. As always, real-world models will make scale factor come alive for students and cause them to retain it longer when models are used. Circles, triangle similarity, volume, surface area, and special angles are all under the umbrella of geometry; however, a priority should be given to scale factors as part of the progression of ratio and proportion through middle school.

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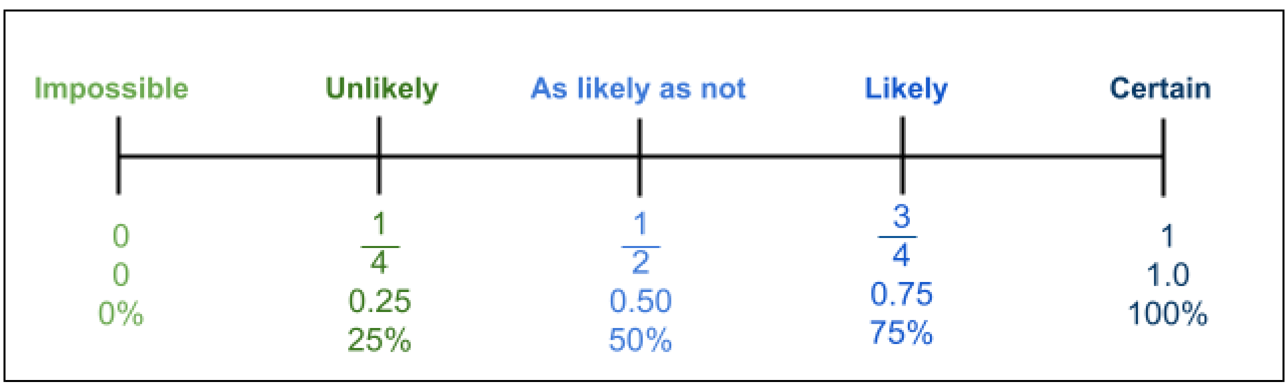
Data

The Big Math Idea of data in 7th grade focuses on sample size, populations, and data distributions, and these ideas are foundational to future explorations into probability and statistics in later courses. This content is another area where it is a medium level standard for ILEARN but a **high level standard** for middle school mathematics development. Often teachers save data and statistics until the end of the year, and only cover it if they have time. This could not be more problematic. We recommend data be taught in January at the beginning of the second semester so students have time to work with and understand the concepts.

A conceptual definition of probability should be emphasized in seventh grade, teaching students probability is a value between 0 and 1. Focus on what probability is and what probability tells us about a situation using real-world models.



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Seventh Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Algebra and Functions	**7.AF.1:** Apply the properties of operations (e.g., identity, inverse, commutative, associative, distributive properties) to create equivalent linear expressions, including situations that involve factoring (e.g., given 2x - 10, create an equivalent expression 2(x - 5)). Justify each step in the process.	High	Students can create equivalent linear expressions by applying the properties of operations. Students can explain each step using the properties.
	7.AF.2: Solve equations of the form px + q= r and p(x + q) = r fluently, where p, q, and r are specific rational numbers. Represent real-world problems using equations of these forms and solve such problems.	High	Students can represent real-world situations with two-step equations and solve them.
	7.AF.3: Solve inequalities of the form px +q (> or ≥) r or px + q (< or ≤) r, where p, q, and r are specific rational numbers. Represent real-world problems using inequalities of these forms and solve such problems. Graph the solution set of the inequality and interpret it in the context of the problem.	High	Students can represent real-world situations with two-step inequalities and solve them. Students can graph the solution set and interpret it in the context of the problem.
	7.AF.4: Define slope as vertical change for each unit of horizontal change and recognize that a constant rate of change or constant slope describes a linear function. Identify and describe situations with constant or varying rates of change.	High	Understand slope conceptually and be able to describe situations with and without a constant rate of change. This is NOT using a formula to find slope.
	7.AF.5: Graph a line given its slope and a point on the line. Find the slope of a line given its graph	High	Students can move fluently between numerical and graphical representations of slope.
	7.AF.6: Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).	Medium	Students can test for a proportional relationship using a variety of methods such as tables or graphing.

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	7.AF.7: Identify the unit rate or constant of proportionality in tables, graphs, equations, and verbal descriptions of proportional relationships.	High	Demonstrate the ability to find the unit rate from different representations.
	7.AF.8: Explain what the coordinates of a point on the graph of a proportional relationship mean in terms of the situation, with special attention to the points (0, 0) and (1,r), where r is the unit rate.	Low	Interpret the points on the graph of a proportional relationship, especially the origin and (1,r) where r is the unit rate.
	7.AF.9: Identify real-world and other mathematical situations that involve proportional relationships. Write equations and draw graphs to represent proportional relationships and recognize that these situations are described by a linear function in the form y = mx, where the unit rate, m, is the slope of the line.	High	Write equations of the form y=mx and draw graphs of real-world situations. Understand that m is the unit rate of the problem and the slope of the line.
	7.DSP.1: Understand that statistics can be used to gain information about a population by examining a sample of the population and generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences	Medium	Understand the relationship between samples and the total population. Explain that random sampling tends to produce samples that are representative of the population.
	7.DSP.2: Use data from a random sample to draw inferences about a population. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.	Medium	Students can draw inferences about a population given data from random samples.
	7.DSP.3: Find, use, and interpret measures of center (mean and median) and measures of spread (range, interquartile range, and mean absolute deviation) for numerical data from random samples to draw comparative inferences about two populations.	High	Demonstrate the ability to find, use, and interpret measures of center and spread for two populations and draw inferences from the data.
Data Analysis, Statistics and	**7.DSP.4:** Make observations about the degree of visual overlap of two numerical data distributions represented in line plots or box plots. Describe how data, particularly outliers, added to a data set may affect the mean and/or median.	Medium	Compare data distributions looking at the visual overlap of the two sets. Students can describe how adding data effects the mean or median.

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Probability	**7.DSP.5:** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Understand that a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. Understand that a probability of 1 indicates an event certain to occur and a probability of 0 indicates an event impossible to occur.	High	Students will understand probability conceptually. Students can explain that it is a number between 0 and 1, and how likely or unlikely an event is.
	7.DSP.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its relative frequency from a large sample.	Medium	Understand that the experimental probability of an event is based on collected data and be able to approximate the probability of the event happening.
	7.DSP.7: Develop probability models that include the sample space and probabilities of outcomes to represent simple events with equally likely outcomes. Predict the approximate relative frequency of the event based on the model. Compare probabilities from the model to observed frequencies; evaluate the level of agreement and explain possible sources of discrepancy.	Medium	Students can explain the difference between theoretical and experimental probability using a model.
	7.GM.1: Draw triangles (freehand, with ruler and protractor, and using technology) with given conditions from three measures of angles or sides, and notice when the conditions determine a unique triangle, more than one triangle, or no triangle.	Medium	Students can decide what conditions create a unique triangle, more than one triangle, or no triangle. Emphasis should be given to using technology.
	7.GM.2: Identify and describe similarity relationships of polygons including the angle-angle criterion for similar triangles, and solve problems involving similarity.	Medium	Solve real-world problems using similarity.
	7.GM.3: Solve real-world and other mathematical problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing. Create a scale drawing by using proportional reasoning.	High	Solve real-world problems involving scale drawings.

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Geometry and Measurement	**7.GM.4:** Solve real-world and other mathematical problems that involve vertical, adjacent, complementary, and supplementary angles.	Medium	Students know vertical, adjacent, complementary, and supplementary angles and use that knowledge to solve real-world problems.
	7.GM.5: Understand the formulas for area and circumference of a circle and use them to solve real-world and other mathematical problems; give an informal derivation of the relationship between circumference and area of a circle.	High	Students can explain how area and circumference of a circle are related. Students can informally describe how the formulas for area and circumference are derived. Students will use the formulas (without memorization) to solve real-world problems.
	7.GM.6: Solve real-world and other mathematical problems involving volume of cylinders and three-dimensional objects composed of right rectangular prisms.	Medium	Solve *real-world problems* using volume of cylinders and 3D objects composed of right rectangular prisms.
	7.GM.7: Construct nets for right rectangular prisms and cylinders and use the nets to compute the surface area; apply this technique to solve real-world and other mathematical problems.	Low	Solve *real-world problems* involving surface area of cylinders and right rectangular prisms using their nets.
	7.C.1: Understand p + q as the number located a distance |q| from p, in the positive or negative direction, depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	High	Students can add integers and explain the addition of integers using models such as integer chips or number lines, and describe real-world contexts.
	7.C.2: Understand subtraction of rational numbers as adding the additive inverse, p − q = p + (−q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	High	Students can subtract integers and explain the subtraction of integers using models such as integer chips or number lines, and describe real-world contexts.
	7.C.3: Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers.	High	Students can multiply integers and explain the multiplication of integers using models such as integer chips or number lines, and describe real-world contexts.

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Computation	**7.C.4:** Understand that integers can be divided, provided that the divisor is not zero, and that every quotient of integers (with non-zero divisor) is a rational number. Understand that if p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$.	High	Students can divide integers and explain the division of integers using models such as integer chips or number lines, and describe real-world contexts.
	7.C.5: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	High	Students can demonstrate the ability to find unit rates given like or different units in a ratio.
	7.C.6: Use proportional relationships to solve ratio and percent problems with multiple operations, such as the following: simple interest, tax, markups, markdowns, gratuities, commissions, fees, conversions within and across measurement systems, percent increase and decrease, and percent error.	High	Students can apply proportional relationships to solve problems (without focusing on a single, particular method).
	7.C.7: Compute with rational numbers fluently using a standard algorithmic approach.	Medium	Compute with rational numbers. Mastery should focus on real-world situations.
	7.C.8: Solve real-world problems with rational numbers by using one or two operations.	High	Solve real-world problems involving rational numbers with up to two operations.
Number Sense	**7.NS.1:** Find the prime factorization of whole numbers and write the results using exponents.	Low	Show them how to do a factor tree, practice it a few times, then move on.
	7.NS.2: Understand the inverse relationship between squaring and finding the square root of a perfect square integer. Find square roots of perfect square integers.	Medium	Demonstrate an understanding of the relationship between squaring and find the square root. Students can find square roots of perfect squares <u>only</u>.
	7.NS.3: Know there are rational and irrational numbers. Identify, compare, and order rational and common irrational numbers ($\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π) and plot them on a number line.	High	Demonstrate an understanding of rational and irrational numbers by identifying, comparing, and ordering them on a number line.

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Big Math Ideas

8th Grade

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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with

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May 2022

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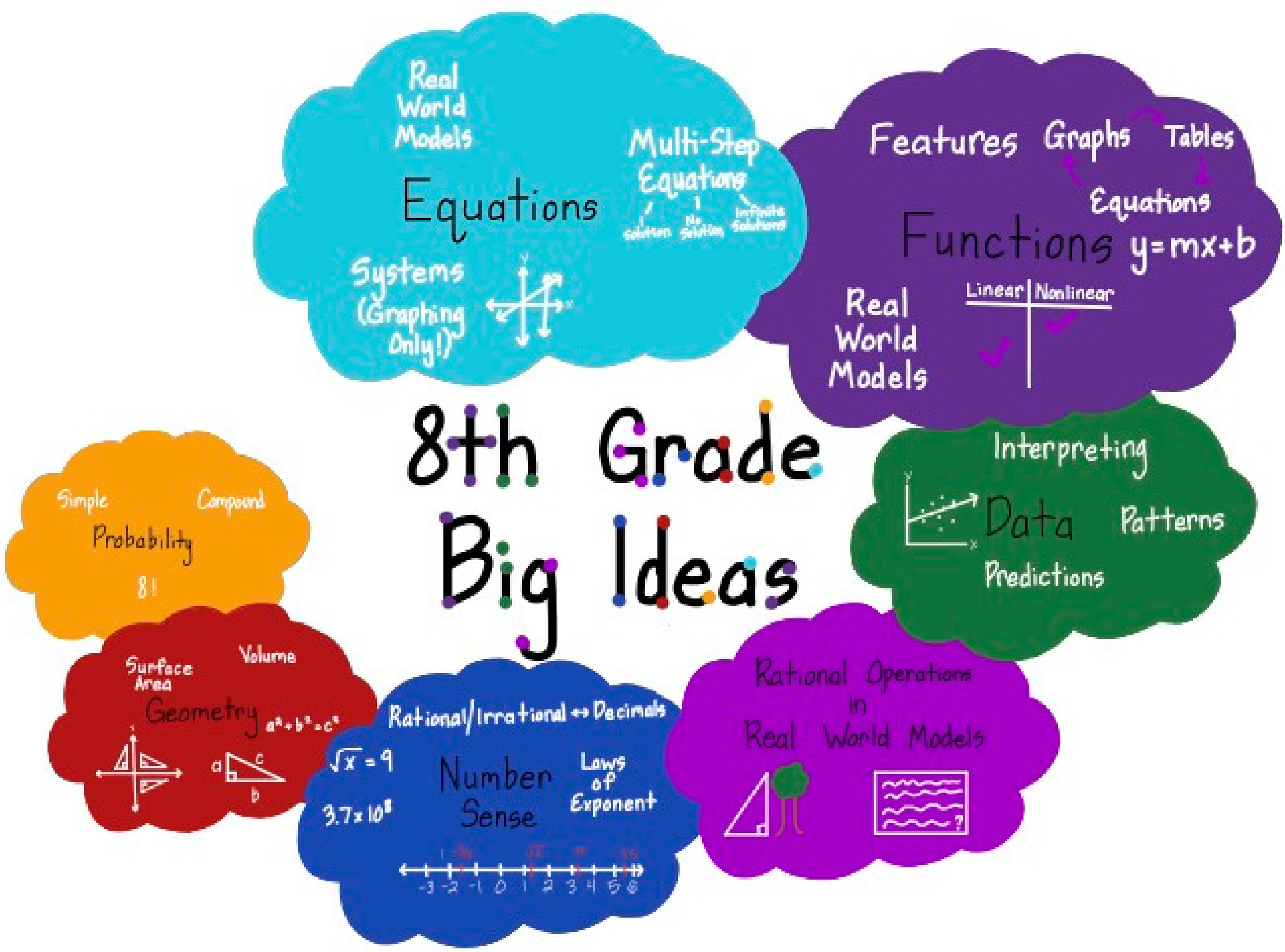
8th Grade Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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8th Grade Big Math Ideas - Narrative

The following narrative will guide you through the Big Math Ideas diagram above. Visual modeling and use of real-world models of concepts cannot be overemphasized, even when you think your students are catching on quickly and don't need it. The conceptual understanding modeling gives students is important for their future development as mathematicians.

Functions

The introduction of functions is a major area of focus for 8th grade. The concept of functions grows out of the ratio work in 6th grade and the proportionality work in 7th grade. We continue the concept of rate of change and extend $y = kx$ to $y = mx + b$. The constant of proportionality in 7th grade now becomes *the rate of change* and *slope* because we have a y-intercept in our real-world models and we want students to begin to equate rate of change to slope of a line. Students compare linear and nonlinear models through an examination of the rate of change and behavior in a table and a graph. We describe the features of a function such as: where it is increasing and decreasing, linear or nonlinear, and domain and range. Throughout this work in functions, it is important students see functions through multiple representations such as tables, graphs, equations, and real-world models. This helps them understand functions conceptually.

Data

Data in 8th grade connects to the linear functions standards through scatterplots. This includes constructing and interpreting scatterplots and describing patterns such as clustering, outliers, and associations that are positive or negative and linear or nonlinear. Students will also need to informally fit a straight line to the data in a scatterplot, write an equation for that line, and use the equation to make predictions about what is happening inside and outside the data. These are all important concepts and should be modeled heavily by students using technology and real-world situations. Data should be taught at the beginning of January, not

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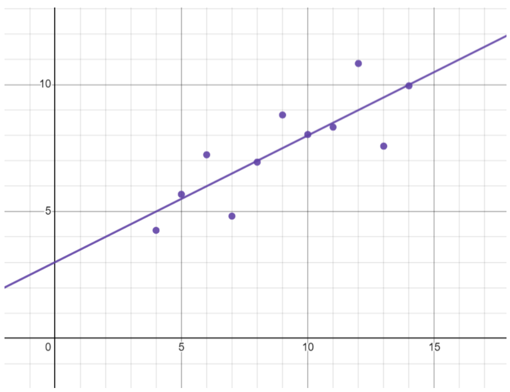
toward the end of the school year, so students have time to work with and understand the concepts. Too often teachers save this topic until May because they think it is less important and can wait, then they run out of time and rush through it. Data is an important concept and should be given a sufficient amount of time for students to master it.



Probability will include finding simple and compound probabilities as well as a firm understanding of the counting principle and its applications. Real world models should be used extensively in this topic.

Equations

Work with equations continues to be a Big Math Idea in 8th grade moving from two step equations in 7th grade to multi-step equations that have one solution, no solution, or infinite solutions in 8th grade. It is important not to start back with one-step equations. If students are struggling with solving basic equations, they will gain practice as you dive into multi-step equations. You can support and intervene for individual students as needed. Modeling with manipulatives like algebra tiles is important, even if you feel some of your students don't need it because they can use the algorithm of inverse operations successfully. Real-world models are always a focus when working on equations. Students should be solving equations with tables and graphs and using technology such as Desmos. Although students are solving multi-step equations in 8th grade, the variable is always on only one side of the equation. It is critical that students are not asked to engage with page after page of equations to solve. Instead, students should engage with real world problems that

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are solved with equations and students are making sense of the solution in the context of the situation.

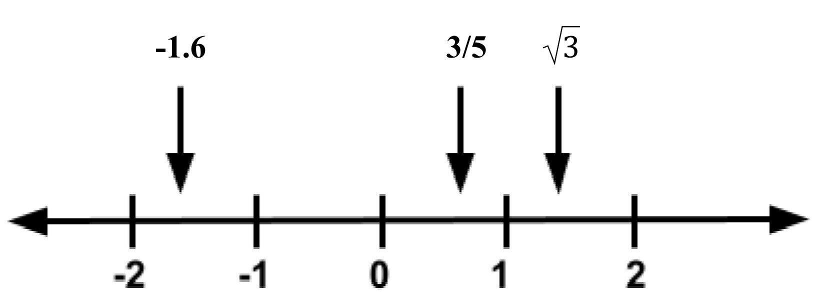
Writing Equations to Model Real-world Situations

A tuxedo rental service charges a $125 flat fee for a suit plus $40 per additional day. Write an equation to model the total cost of renting a tuxedo for x number of days. What would the total cost be if you wanted to rent a tuxedo for 5 days?

Systems of Equations is also introduced in 8th grade with an emphasis on solving by *graphing only*. Desmos can and should be utilized to help students to conceptually understand the solution to a system of linear equations is an ordered pair. Students need to engage with real world models that represent a system of equations as well. The key concept in teaching systems of equations in 8th grade is that students understand where the lines intersect is the solution to the system. The use of technology is vital in teaching this concept.

Number Sense

Number sense encompasses an important set of standards in 8th grade including the laws of exponents, comparing irrational numbers, and solving real-world problems with rational numbers. It is important that students discover and develop the laws of exponents through expanded form rather than just memorizing them. We give special emphasis here to understanding the relationship between rational and irrational numbers and their place on a number line, as well as being able to change rational and irrational numbers to decimals for comparison.



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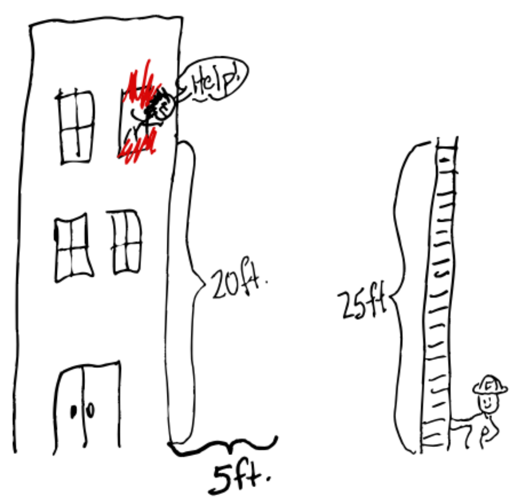
Operations with rational numbers is also a high priority in eighth grade and should be taught using real-world models. Using concrete models give these abstract ideas stickiness allowing the knowledge to be retained longer and transferred to other situations

Geometry

The two most important concepts in 8th grade geometry are the Pythagorean Theorem (what it is, how it was discovered, and how it is used) and transformations. There are other standards to teach, but they are less critical. Other concepts under the geometry umbrella at this level include volume, surface area, and slices of 3D figures with real-world models being used throughout the lessons.



Imagine that you're a firefighter. There is a person in a burning building on the 3rd floor. The ladder must be 5 ft from the base of the building for safety. Your ladder is 25 ft. long. Will your ladder reach the person?

Using real-world models and manipulatives as much as possible will give students the conceptual understanding they need for future success as mathematicians.

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Eighth Grade Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Algebra and Functions	**8.AF.1:** Solve linear equations with rational number coefficients fluently, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Represent real-world problems using linear equations and inequalities in one variable and solve such problems.	High	Students can solve linear equations with variables on one side only. Students can write and solve linear equations and inequalities in one variable from real-world situations.
	8.AF.2: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by transforming a given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).	High	Students can decide whether an equation has one solution, infinitely many solutions, or no solutions.
	8.AF.3: Understand that a function assigns to each x-value (independent variable) exactly one y-value (dependent variable), and that the graph of a function is the set of ordered pairs (x,y).	Medium	Students can explain what a function is in terms such as input and output, x-values and y-values, and independent and dependent variables.
	8.AF.4: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, has a maximum or minimum value). Sketch a graph that exhibits the qualitative features of a function that has been verbally described.	High	Describe a graph qualitatively in a variety of ways and sketch a graph given a verbal description.
	8.AF.5: Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. Describe similarities and differences between linear and nonlinear functions from tables, graphs, verbal descriptions, and equations.	High	Understand and interpret the equation y=mx + b. Describe the similiarities between linear and nonlinear functions from tables, graphs, verbal descriptions, and equations.
	8.AF.6: Construct a function to model a linear relationship between two quantities given a verbal description, table of values, or graph. Recognize in y = mx + b that m is the slope (rate of change) and b is the y-intercept of the graph, and describe the meaning of each in the context of a problem.	High	Given a variety of representations, construct a function to model the relationship shown. Interpret the rate of change and the y-intercept in the context of a problem.

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	8.AF.7: Compare properties of two linear functions given in different forms, such as a table of values, equation, verbal description, and graph (e.g., compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed).	Medium	Compare properties of linear functions in different representations.
	8.AF.8: Understand that solutions to a system of two linear equations correspond to points of intersection of their graphs because points of intersection satisfy both equations simultaneously. Approximate the solution of a system of equations by graphing and interpreting the reasonableness of the approximation.	Medium	Solve a system of linear equations by graphing *only*. Students should NOT be solving by substitution or elimination.
Data Analysis, Statistics and Probability	**8.DSP.1:** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantitative variables. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	High	Use technology to construct and interpret scatter plots. Students can describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
	8.DSP.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and describe the model fit by judging the closeness of the data points to the line.	Medium	Understand what a line of fit is that models the relationship between two quantitative variables.
	8.DSP.3: Write and use equations that model linear relationships to make predictions, including interpolation and extrapolation, in real-world situations involving bivariate measurement data; interpret the slope and y-intercept.	High	Write and equation for the line of fit and use it to make predictions in real-world situations. Interpret the slope and y-intercept of the line of fit.
	8.DSP.4: Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Understand and use appropriate terminology to describe independent, dependent, complementary, and mutually exclusive events.	Medium	Understand and describe probability of compound events including independent, dependent, complementary, and mutually exclusive events.
	8.DSP.5: Represent sample spaces and find probabilities of compound events (independent and dependent) using methods, such as organized lists, tables, and tree diagrams.	Medium	Find probabilities of compound events using a variety of methods such as organized lists, tables, and tree diagrams.

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	8.DSP.6: For events with a large number of outcomes, understand the use of the multiplication counting principle. Develop the multiplication counting principle and apply it to situations with a large number of outcomes.	Medium	Develop and understand the counting principle. Students can use the counting principle to describe situations with a large number of outcomes.
Geometry and Measurement	**8.GM.1:** Identify, define and describe attributes of three-dimensional geometric objects (right rectangular prisms, cylinders, cones, spheres, and pyramids). Explore the effects of slicing these objects using appropriate technology and describe the two-dimensional figure that results.	Medium	Describe attributes of 3D geometric objects. Students can describe the 2D results of slicing 3D objects using technology.
	8.GM.2: Solve real-world and other mathematical problems involving volume of cones, spheres, and pyramids and surface area of spheres.	High	Solve real-world problems involving volume of cones, spheres, and pyramids and surface area of spheres *without memorizing the formulas.*
	8.GM.3: Verify experimentally the properties of rotations, reflections, and translations, including: lines are mapped to lines, and line segments to line segments of the same length; angles are mapped to angles of the same measure; and parallel lines are mapped to parallel lines.	Medium	Use technology to experiment with rotations, reflections, and translations, showing that they each preserve size and shape.
	8.GM.4: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Describe a sequence that exhibits the congruence between two given congruent figures.	Medium	Students can describe a sequence of rotations, reflections, and translations to move a pre-image to its congruent image. This should be done using technology.
	8.GM.5: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Describe a sequence that exhibits the similarity between two given similar figures.	Medium	Students can describe a sequence of rotations, reflections, and translations and diliations to obtain a similar image from its pre-image. This should be done using technology.
	8.GM.6: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates	Medium	Describe what happens to the coordinates of a figure during a particular transformation on the coordinate plane.
	8.GM.7: Use inductive reasoning to explain the Pythagorean relationship.	Medium	Use a model to explain the Pythagorean relationship.
	8.GM.8: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and other mathematical problems in two dimensions.	High	Use the Pythagorean Theorem to solve real-world problems.
	8.GM.9: Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane.	High	Use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

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Computation	**8.C.1:** Solve real-world problems with rational numbers by using multiple operations.	High	Solve real-world problems with rational numbers.
	8.C.2: Solve real-world and other mathematical problems involving numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Interpret scientific notation that has been generated by technology, such as a scientific calculator, graphing calculator, or excel spreadsheet.	Low	Investigate real-world problems involving scientific notation.
Number Sense	**8.NS.1**: Give examples of rational and irrational numbers and explain the difference between them. Understand that every number has a decimal expansion; for rational numbers, show that the decimal expansion terminates or repeats, and convert a decimal expansion that repeats into a rational number.	Medium	Demonstrate an understanding of rational and irrational numbers by explaining the difference between them.
	8.NS.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, plot them approximately on a number line, and estimate the value of expressions involving irrational numbers.	High	Students can estimate the size of irrational numbers to compare them and plot them on a number line.
	8.NS.3: Given a numeric expression with common rational number bases and integer exponents, apply the properties of exponents to generate equivalent expressions.	High	Simplify numeric expressions with integer exponents using the properties of exponents.
	8.NS.4: Use square root symbols to represent solutions to equations of the form x^2 = p, where p is a positive rational number.	Medium	Use square roots to solve equations of the form x^2 = p.

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High School Mathematics

Why the need to change?

The need for change in the second decade of the 21st century is urgent. Our high school students do not think of themselves as mathematicians. They generally don't see the application of the mathematics they learn in school or in their world, and they don't enjoy learning the mathematics we teach them in the way we teach them (Boaler, 2019). As a result, we have a significant shortage of American college students majoring in mathematics, data, computer science, and STEM related fields, which means our American companies outsource these jobs to individuals from other countries. Resultantly, our students are not majoring in fields of study where the jobs are available because of their mathematical mindsets.

Additionally, we have a calculus problem. In Indiana, our required high school math course of study and the pedagogical focus in those courses, namely Algebra 1, Geometry, and Algebra 2, mathematically assume we are preparing every student to take Calculus. In actuality, 10-16% of college freshmen are required to take Calculus as a part of their course of study for their major (Boaler, 2019). In fact, our Indiana state universities required the following courses for approximately 85-90% of the freshman class in 2021: Finite Mathematics, Quantitative Reasoning, and Discrete Mathematics. In these courses, students study statistics, set theory, probability, matrices, and other topics we do not teach in our current high school Core 40 courses to any discernible, effective depth. We need to ask ourselves why we are preparing all of our students for a 4 year college, Calculus required program, instead of preparing them for a wide variety of college and career experiences.

The "Big Math Ideas" for high school mathematics are based primarily on the recommendations and guidance of NCTM's *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* as well as the references listed below. I encourage a deep dive into these references and reflection on what mathematics we are teaching and how we are teaching it in our current high school courses. Our students deserve a different kind of mathematics - mathematics which meets the needs for their future. The rote memorization of

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countless, unconnected procedures is a 1980's rite of passage which has far overstayed its welcome. The students of today can Google the answer to any one of a variety of procedural questions in seconds, as any high school teacher will attest . We want students who can think critically and problem solve through tasks that aren't "Googleable." Employers want to hire employees who can solve new and unique problems, work collaboratively, innovate and work creatively, and think outside the box. They do not want employees who can memorize procedures then mindlessly replicate what is told to them without any real understanding of what they are doing.

The Indiana Academic Standards shifted in 2020; yet the real change began in 2014 when the standards became college and career ready standards. Unfortunately, many educators and textbook publishers have been interpreting the standards as a set of procedures to be memorized. The following "Big Math Ideas" are the Indiana Academic Standards and the college and career ready standards and the shift called upon by NCTM combined into one document. They are the how, the what, AND the why of high school mathematics.

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Big Math Ideas

Algebra 1

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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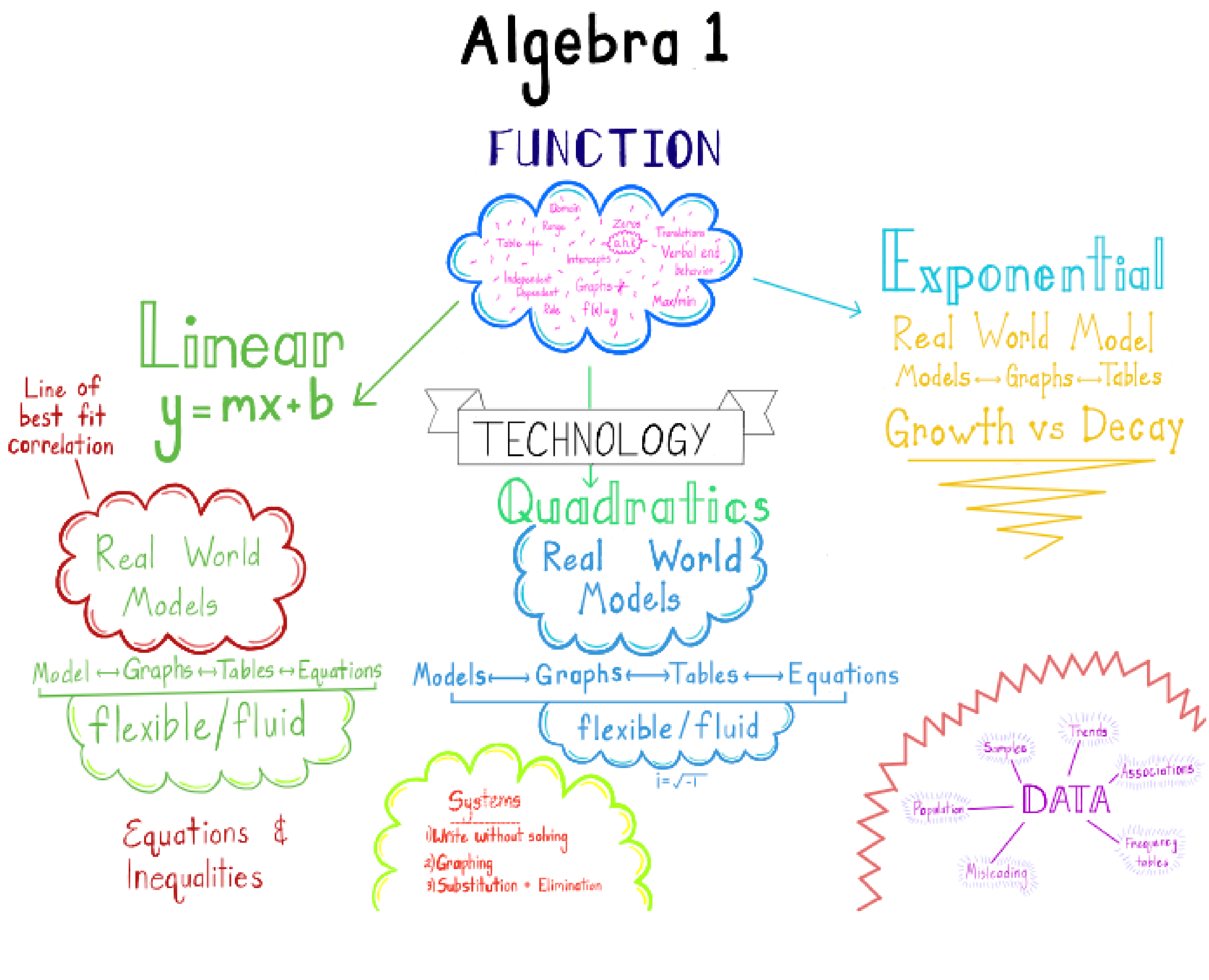
Algebra 1 Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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Algebra 1 Big Math Ideas - Narrative

Algebra 1 is the most transformed course in the high school sequence and teaching it will feel very different than it has in the past. The Algebra 1 of the past was about teaching students how to solve various types of equations (mostly linear and quadratic) using algebraic manipulation. The 2022 version of Algebra 1 is focused on showing students how real-world situations are modeled using functions, systems of equations, and data.

Diving into Functions

The major focus of the Algebra 1 course is the very important concept of functions, the characteristics of functions (domain, range, max/min, intercepts, independent, dependent, rate of change, etc.), types of functions that fit into Algebra 1 (linear, quadratic, exponential), the 4 representations of functions (tables, graphs, equations, verbal models), and the real-world situations that are modeled by these mathematical functions.

Algebra 1 Functions

Type of Function	**Characteristics**	**Representation of the functions**	**Priority in Algebra**
Linear	Domain, range, intercepts, independent, dependent, rate of change, slope, increasing, decreasing, real world model vs function rule, parent function, other forms of the	Fluidly and flexibly move between these 4 representations. Students can be given 1 representation and find the other 3. Table, graph, equation f(x),	High - Mastery required

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	function equation.	real-world model	
Quadratic	Domain, range, intercepts, zeros, solutions, independent, dependent, rate of change (1st and 2nd differences in a table) increasing, decreasing, real world model vs function rule, max/min, parent function	Fluidly and flexibly move between these 4 representations. Students can be given 1 representation and find the other 3. Table, graph, equation f(x), real world model	High - Mastery of what real-world situations model a quadratic function, and characteristics of a quadratic function.
Exponential	Domain, range, intercepts, independent, dependent, increasing, decreasing, real world model, max/min, f(x), parent function	Table, graph, equation f(x), real world model Students should be able to work with all 4 representations.	Medium - Explore real-world situations for growth and decay, identify the f(x) and graph.

Functions are first introduced in 8th grade and are compared to relations. The concept of 1:1 correspondence is introduced in 8th grade as is the definition of domain and range. We should begin our discussion of functions in Algebra 1 with **linear functions** very early in the school year. Students should be introduced to how to represent real-world models of linear functions with tables, graphs, equations (f(x) = y), and real world models. Students should be able to fluidly and flexibly solve and work with any of these representations of linear functions. This means students should be able to, from any of these representations, be able to derive the other 3 representations. Take advantage of Desmos to support students learning the

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relationships between a function's representations. When we always make students calculate a table or graph by hand, the student can spend so much time completing these hand calculations they are checked out by the time the big 'ah ha' moment arrives. Give the students technology when you want them to see the modeling. This Desmos activity is a great example of how to use this tool to enhance the conceptual understanding for the students.

Linear functions should be referred to as linear functions, not linear equations. Function notation f(x) should be used as often as possible. Terminology such as "rate of change" and "slope" should be used interchangeably. Terminology such as "rise over run" should be avoided as it often detracts from a student's conceptual understanding of slope/rate of change. Students should be able to find the slope of a linear function from a graph, table or function rule (equation) without using the formula. The most important form of the equation of the line is slope-intercept form. Standard form and point-slope form should be briefly discussed as other forms of a linear function; however, slope-intercept form can always be used with any given information. Students should not be formally assessed on their ability to algebraically convert a linear function from one form of the equation to another. It is more important that students can explain the pros and cons of each form of the equation. It is critical students are able to move fluidly and flexibly within all 4 representations of the linear function: the real world model, table, graph, and equation (function rule). This Illustrative Mathematics task is an excellent example of the type of task students should be able to do fluidly and flexibly within the unit on linear functions. This task is also a great model for the type of assessment questions needed in the Algebra 1 course. Students should have a mastery level of understanding for linear functions by the end of the year. The instructional time spent on linear functions is NOT spent on solving linear equations, this is a separate standard all together.

Quadratic functions should be taught in a very similar way to linear functions. Educators should introduce quadratic functions by introducing real-world situations which model quadratic functions such as throwing a ball into the air, kicking a soccer ball, dropping an object from a building, and the trajectory of something shot from a cannon (i.e. a potato launcher), then demonstrating with technology (Desmos) how

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the data, usually time vs. height of the object, appears on a graph. Students will begin to see, the data from these situations model the same shaped graph. We can then introduce them to a quadratic function with the exact same characteristics as linear functions (domain, range, independent, dependent, max/min, intercepts, etc.) with 4 different representations (table, graph, equation, real-world model) where the students need to be able to fluidly and flexibly move between the representations.

You may be asking yourself, ``When do I spend 3 weeks factoring quadratic equations?" You don't! We want students to understand quadratic functions, and, yes, quadratic equations are **one** of the representations of quadratic functions. Students will learn to solve quadratic equations; however, this is a small segment of the work they will be doing with quadratic functions. There are several ways to solve a quadratic equation, and students need to understand what they are solving the equation for before they learn to solve it. As a result, after spending a substantial amount of time on modeling situations with quadratic functions where students gain a mastery understanding of real-world problems that create quadratic functions, students will learn to solve quadratic equations.

Students begin to solve quadratic equations by first defining the vocabulary of polynomials (monomials, binomials, trinomials, degrees) then move to **adding and subtracting polynomial expressions.** Students should be tasked with adding and subtracting polynomial expressions vertically and horizontally to demonstrate various strategies. Multiplying and dividing polynomial expressions should follow addition and subtraction. We want multiplication and division to follow the real-world modeling the students have seen with quadratic functions. Therefore, using the concept of area for multiplication not only provides a real-world connection for students, it supports a conceptual understanding for all students. The CUNY HSE Curriculum Lesson provides a wide range of ideas for supporting your students with multiplication of integers to multiplication of polynomial expressions using the area model. The use of the area model rather than the FOIL method can have a substantially positive effect on a student's mathematical understanding of both multiplication and factoring of quadratic expressions and hence solving quadratic equations. After multiplying using the area model, move to division of polynomial expressions by a monomial (A1.NE.5).

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Once the students have mastered addition, subtraction, multiplication and division with polynomial expressions, move back into quadratic functions and quadratic equations as students begin to discover the need to factor trinomials. By returning to quadratic functions and the real world models associated with them, we are answering questions such as:

1. When does the ball hit the ground?
2. How long is the soccer ball in the air?
3. When I drop the water balloon from the top of the slide, how long does it take to reach the ground?
4. What speed was the car going when it tried to stop before the accident?

Students now have a reason to learn to factor the quadratic equations, or use the quadratic formula, or use technology to solve by graphing. To teach students to factor quadratic equations, use the area model shared above. It is not important to provide multiple problems where the GCF needs to be factored out, or specifically over emphasize special products, instead it is more important students conceptually understand they are finding the intercepts of a quadratic function, the zeros of quadratic equation, and the solution to a quadratic equation. They need to demonstrate their understanding and explain what those zeros (solutions, intercepts) mean in the context of the real world model.

Sample Task: This task could be factored or solved using technology to graph for part 3.

A toy rocket is fired into the air from the top of a barn. Its height (h) above the ground in yards after t seconds is given by the function $h(t)=-5t^2+10t+20$

1. *What was the maximum height of the rocket?*
2. *How long was the rocket in the air before hitting the ground?*
3. *At what time(s) will the rocket be at a height of 22 yd?*

The last function we explore in Algebra 1 is **exponential functions.** Students are not expected to master exponential functions in Algebra 1; they will do this in

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Algebra 2. Educators should introduce exponential functions similarly to linear and quadratic functions, with real-world situations that model growth and decay. Start with a real-world situation that resonates with your students, not a stuffy interest rate task. Here are some examples of exponential growth and decay functions you could utilize as models to introduce these functions.

Exponential decay: The NCAA Basketball Championship (also known as March Madness) is an example of exponential decay. At each round of the tournament, teams play against one another with only the winning teams progressing to the next round. In other words, the number of teams playing at each round is half of the number of teams playing in the previous round. Let's start with 64 teams going into round 1. How many teams are left to begin play in round 5? (Copyright © 2012-2022 MathBitsNotebook.com)

Exponential growth: A diamond ring was purchased 20 years ago for $500. The value of the ring increased by 3% each year. What is the ring worth today?

Exponential functions are just like all the other functions students have studied all year: they have all the characteristics of functions (domain, range, independent, dependent, intercepts, etc.) and 4 representations (graphs, tables, equations, real world models). However, there are limits to what you will have students do with exponential functions in Algebra 1. They will graph the functions using technology, and they will be able to build a table of values (with and without technology). Students will be able to identify an equation but will NOT solve the equations by hand; however, they will be able to determine a solution to a real-world problem using technology.

By the end of Algebra 1 students will have a mastery level understanding of functions, the characteristics of functions in general, the 4 representations of functions (graph, table, equations, real world models) and will be able to apply this knowledge to new types of functions they will learn in Algebra 2. The concept of functions is THE most important content taught in Algebra 1.

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Systems of Linear Equations

Students have been introduced to systems of equations in 8th grade and have solved systems of equations by graphing only. However, students are not expected to master solving by graphing in 8th grade, but they are expected to have mastery of systems of linear equations by the end of Algebra 1. Begin the unit on systems of equations by asking students to write the system of equations from real-world situations ONLY. They should not solve any actual systems until they can fluidly and flexibly write the system of equations from real world situations. Be sure the real world tasks used are relevant and relational to your students.

Begin solving the system of equations the students have written in the first part of the unit by graphing with and without technology. The purpose is for the students to fully understand they are looking for a point where the two lines intersect, which produces an ordered pair answer. The students should always be able to explain their answers in context of the real-world situation. Limit the number of problems assigned to students which are not connected to a real-world model.

After the students have a clear conceptual understanding of what they are solving for then, move into either substitution or elimination, then the other solution method. Students only have to master ONE way to solve a system of equations. They do not need to demonstrate mastery of all three solution methods. Therefore, demonstrate all three methods, but students should have a **choice** in how they demonstrate their understanding on both formative and summative assessments. By allowing students to choose their own method of solving a system of equations, the students are demonstrating their understanding of the mathematical concept which demonstrates the necessary level of mastery for Algebra 1.

Students will need to be exposed to graphing a system of linear inequalities as well. The key understanding for the students is there are multiple solutions to the system of inequalities, not just one point. Again, it is critical students are given tasks that model real-world situations and can explain the solutions in the context of the model. This is not a priority standard and should not be taught to mastery.

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Data

Data is the third most important concept taught in Algebra 1. It is highly recommended the data unit is taught as the first unit in 2nd semester. Students need data analysis skills for both college coursework and future employment; yet educators continue to treat it as an "if I get to it" concept. Students cannot afford for us to continue to take this approach. The data unit will include three standards (DS.1, 2, 5). The students will need a strong understanding of a sample vs. a population and how a random sample can represent a population in an experiment, observation sample, or sample survey. In addition, students will understand two-way frequency tables as a way to examine trends in bivariate categorical data. Finally, students will learn how statistical data is non-neutral and can be used to support or defend a specific interest, hence introducing the idea of bias in data representation. This unit is an opportunity to open students' minds to a world of mathematics they have had limited exposure to, yet often they find it interesting and fun. Dig in as there are so many possible projects, activities, and fascinating applications to engage students' minds.

The other 2 data standards (DS. 3 and 4) will be taught during the linear functions unit as they apply to linear correlation and scatter plots that fit linear models.

Linear Equations/Inequalities

Although linear equations and inequalities are **not** major concepts for Algebra 1, most teachers believe it is important; therefore we will spend a moment addressing this misconception. Refer to the Middle School Expressions/Equations Learning Progressions for a complete explanation of how and when students develop their mathematical understanding of linear equations. It is important to note students solve one step equations in 6th grade, two step equations in 7th grade and multi-step linear equations in 8th grade (with the variable on one side of the equation).

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6th grade	7th grade	8th grade
$x + 6 = 10$ $2x = 12$ $\frac{x}{5} = 4$	$-3x + 9 = 6$ $\frac{x}{4} - 10 =- 2$ $5x + 7 = 12$	$2x - 8 - 5x = 15$ $4x - (8x - 9) = 11$ $-6c + \frac{c}{4} - \frac{1}{2} = 8$

Resultantly, the only standard to teach in Algebra 1 related to linear equations and inequalities is A1.L1 which references solving **real-world problems** which model linear equations or inequalities. This means educators should **not** spend 3 weeks at the beginning of the school year reviewing all the way back to one-step equations because students tell us they don't remember ever solving equations. We need to begin the year by jumping into real-world situations that model all levels of linear equations. The students should be able to write and solve the corresponding linear equations. The focus is on writing the equation from the situation, solving the equation, then making sense of the solution within the context of the situation. Page after page of equations for students to practice solving is NOT this Algebra 1 standard. These are real-world situations, the equations and inequalities have variables on **both sides** of the equation and students are able to justify, using algebraic properties they have learned throughout middle school, how they are able to solve the equation/inequality. This work with linear equations and inequalities should not take more than 5 instructional days at the beginning of the year. This is **not** a Big Math Idea for Algebra 1. If students do not have mastery of linear equations/inequalities initially, they will continue to work with linear functions for most of the first semester to support their development towards mastery of linear equations.

Minor concepts of Algebra 1

- Solving/graphing real-world linear inequalities in two variables. Sample task
- Literal equations. Sample: I = PRT solve for R
- Compound linear inequalities. Sample from Math Planet
- Introduce i = $\sqrt{-1}$ during your introduction of the quadratic formula.

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- Simplify rational expressions where the numerator and denominator are monomials. Students are applying the laws of exponents. Laws of exponents are mastered in 8th grade. Sample video
- Simplify square roots of non-perfect square monomial expressions. $\sqrt{48x^2y^5}$

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Algebra 1 Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Data Analysis and Statistics	**AI.DS.1** Understand statistics as a process for making inferences about a population based on a random sample from that population. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Medium	Students will conceptually understand the meaning of a sample as a statistical representation of a population. Students will understand and conduct the various types of samples utilized to represent a population in **real-world situations.**
	AI.DS.2 Understand that statistics and data are non-neutral and designed to serve a particular interest. Analyze the possibilities for whose interest might be served and how the representations might be misleading.	High	Students will understand, using real-world situations, how statistics can be used in non-neutral ways to influence decisions.
	AI.DS.3 Use technology to find a linear function that models a relationship between two quantitative variables to make predictions, and interpret the slope and y-intercept. Using technology, compute and interpret the correlation coefficient.	High	Students will understand a linear function can be used to model the relationship between the quantities when given two quantities in a real-world situation. Students will interpret the slope and y-intercept of the model. Students will use technology to find the correlation coefficient, and understand its meaning in the context of the real-world situation and the relationship between the two quantities.
	AI.DS.4 Describe the differences between correlation and causation.	Low	Students will have a cursory knowledge of the difference between causation and correlation.
	AI.DS.5 Summarize bivariate categorical data in two-way frequency tables. Interpret relative frequencies in the contexts of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in data.	Medium	Students will understand how the use of two-way frequency tables can be utilized to determine relationships, patterns and trends in bivariate, categorical data.

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Number Systems and Expressions	**AI.NE.1** Explain the hierarchy and relationships of numbers and sets of numbers within the complex number system. Know that there is an imaginary number, i, such that √(-1) = i. Understand that the imaginary numbers along with the real numbers form the complex number system.	Low	Understand i is defined as √(-1), it is imaginary and not part of the real number system. It is part of the complex number system of (a+bi). This is an exploration standard only.
	AI.NE.2 Simplify algebraic rational expressions, with numerators and denominators containing monomial bases with integer exponents, to equivalent forms.	Medium	Students will simplify rational expressions using algebraic manipulation. The rational expressions are simplistic, with numerators and denominators that have monomials with negative and positive exponents. These rational expressions do not include factoring to simplify.
	AI.NE.3 Simplify square roots of monomial algebraic expressions, including non-perfect squares.	Medium	Students will simplify square roots that include variables with exponents and non-perfect squares.
	AI.NE.4 Factor quadratic expressions (including the difference of two squares, perfect square trinomials and other quadratic expressions).	Medium	Students will factor quadratic expression using the area model or distributive property. Students do not need to memorize special products. Students are not required to factor out the GCF if the expression is otherwise factorable.
	AI.NE.5 Add, subtract, and multiply polynomials. Divide polynomials by monomials.	Medium	Students will add, subtract, and multiply polynomials using various modeling strategies (algebra tiles, drawings, real-world models, area model, distributive property). Students will divide polynomials by monomials using algebraic manipulation and modeling with the above strategies.
	AI.F.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Understand that if f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. Understand the graph of f is the graph of the equation y = f(x) with points of the form (x, f(x)).	High	Students will understand the meaning of a function f(x) as it relates to the relationship between sets (domain, range). Students will create a table, equation, and graph representing f(x) flexibility and fluidity between the 3 representations. Given one representation, students will be able to determine the other 2 representations of the function.
	AI.F.2 Evaluate functions for given elements of its domain, and interpret statements in function notation in terms of a context.	High	Given a real-world situation, students will evaluate domain values and interpret those values in terms of f(x) in the context of the real-world situation.

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Functions	**AI.F.3** Identify the domain and range of relations represented in tables, graphs, verbal descriptions, and equations.	High	Students will flexibly and fluidly identify and understand the domain and range of relations given a table, graph, real-world model and the equation f(x).
	AI.F.4 Describe, qualitatively, the functional relationship between two quantities by analyzing key features of a graph. Sketch a graph that exhibits given key features of a function that has been verbally described, including intercepts, where the function is increasing or decreasing, where the function is positive or negative, and any relative maximum or minimum values, Identify the independent and dependent variables.	High	Students, when give the graph of a function representing a **real-world situation**, can describe the relationship between the independent and dependent variables using the characteristics of a function such as domain/range, intercepts, maximum/minimum, and increasing/decreasing. Students, when given the characteristics of a function representing a **real-world situation**, such as domain/range, intercepts, maximum/minimum, and increasing/decreasing, can sketch the graph of the function.
	AI.L.1 Represent real-world problems using linear equations and inequalities in one variable, including those with rational number coefficients and variables on both sides of the equal sign. Solve them fluently, explaining the process used and justifying the choice of a solution method.	High	Given a **real-world model**, students will solve linear equations and inequalities, including those with variables on both side of the equal sign and rational coefficients. Students will explain their process and justify their solution methods. Students will understand and explain their solution(s) in the context of the real-world model.
	AI.L.2 Solve compound linear inequalities in one variable, and represent and interpret the solution on a number line. Write a compound linear inequality given its number line representation.	Medium	Students will use a number line to solve a compound linear inequality and understand that the solution set of the inequality is represented on the number line. Students can use a variety of methods such as algebra tiles and/or algebraic manipulation to solve these simple compound linear inequalities. Students will, given the number line representation, write the compound linear inequality.

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Linear Equations, Inequalities, and Functions	**AI.L.3** Represent linear functions as graphs from equations (with and without technology), equations from graphs, and equations from tables and other given information (e.g., from a given point on a line and the slope of the line). Find the equation of a line, passing through a given point, that is parallel or perpendicular to a given line.	High	Students will fluidly and flexibly create the graph of a linear function from a table, and an equation f(x), representing a real-world model (with and without technology). Limit problems that are not connected to real-world models. Students understand why a linear function that is parallel has the same slope as the original function slope and a linear function that is perpendicular has a slope that is the negative reciprocal of the original function slope. Students can demonstrate their understanding of the parallel and perpendicular slopes.
	AI.L.4 Represent real-world problems that can be modeled with a linear function using equations, graphs, and tables; translate fluently among these representations, and interpret the slope and intercepts.	High	Students will fluidly and flexibly represent a linear function with a table, graph, and equation f(x), given a real-world model. When provided with one of the three representations, students are able to develop the other two representations of the real-world model. Students can identify and understand the meaning, in the context of the real-world model, of the slope and y-intercept.
	AI.L.5 Translate among equivalent forms of equations for linear functions, including slope-intercept, point-slope, and standard. Recognize that different forms reveal more or less information about a given situation.	Medium	Students understand the information available in of each of the three forms of a linear function equation. Students should be most familiar with the slope intercept form.
	AI.L.6 Represent real-world problems using linear inequalities in two variables and solve such problems; interpret the solution set and determine whether it is reasonable. Graph the solutions to a linear inequality in two variables as a half-plane.	Medium	Students will solve and understand **real-world situations** that model linear inequalities. Students can explain and graph the solution set, as well as explain the reasonableness of their solution.
	AI.L.7 Solve linear and quadratic equations and formulas for a specified variable to highlight a quantity of interest, using the same reasoning as in solving equations.	Low	Students can solve literal equations for an identified variable. This is an exploratory standard only.

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Systems of Linear Equations and Inequalities	**AI.SEI.1** Understand the relationship between a solution of a system of two linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing; approximate solutions when the coordinates of the solution are non-integer numbers.	Medium	Students will understand the solution to a system of linear equations is the intersection of the graphs of the two equations. Students will demonstrate their understanding through graphing with technology and in the context of real-world models.
	AI.SEI.2 Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions, including cases with no solution and infinitely many solutions. Solve systems of two linear equations algebraically using elimination and substitution methods.	Medium	Students will understand the solution to a system of linear equations can be found by using algebraic properties (substitution and elimination methods).
	AI.SEI.3 Write a system of two linear equations in two variables that represents a real-world problem and solve the problem with and without technology. Interpret the solution and determine whether the solution is reasonable.	High	Students will write and solve linear systems of equations that are modeled by real-world situations, while understanding the solutions in the context of the situation and the reasonableness of the solution. Students will solve the systems of equations with and without technology, with an emphasis on conceptual understanding, not on algebraic manipulation.
	AI.SEI.4 Represent real-world problems using a system of two linear inequalities in two variables. Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes with and without technology. Interpret the solution set and determine whether it is reasonable.	Medium	Students will write and solve linear systems of inequalities that are modeled by **real-world situations**, while understanding the solution set in the context of the situation and the reasonableness of the solution set. Students will solve the systems of inequalities with and without technology, with an emphasis on conceptual understanding, not on algebraic manipulation.
	AI.QE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. Understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Compare linear functions and exponential functions that model real-world situations using tables, graphs, and equations.	High	Students will understand and identify real-world situations that model linear functions compared to those that model exponential functions. Students will understand the growth over equal factors vs. over equal intervals of exponential vs. linear functions and be able to create real-world models to describe exponential functions using table, graphs and equations f(x).

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Quadratic and Exponential Equations and Functions	**AI.QE.2** Represent real-world and other mathematical problems that can be modeled with simple exponential functions using tables, graphs, and equations of the form y = abx (for integer values of x > 1, rational values of b > 0 and b ≠ 1) with and without technology; interpret the values of a and b.	Medium	Students will fluidly and flexibly represent exponential functions with tables, graphs, equations f(x), and **real-world models.** Given one of the four representations of exponential functions, students will develop the other three representations of the function with and without technology. These are simple growth and decay exponential functions.
	AI.QE.3 Use area models to develop the concept of completing the square to solve quadratic equations. Explore the relationship between completing the square and the quadratic formula.	Low	Use the area model method to demonstrate to students how we derive the quadratic formula. Students do not need to know how to derive the quadratic formula themselves.
	AI.QE.4 Solve quadratic equations in one variable by inspection (e.g., for x2 = 49), finding square roots, using the quadratic formula, and factoring, as appropriate to the initial form of the equation.	Medium	Students will solve quadratic functions using a variety of strategies including but not limited to: finding the square root, factoring using the area model, distributive property, quadratic formula, graphing with technology. Students will understand what the solution(s) mean in the context of the **real-world model** of the function. Avoid too many problems that lack a real-world model.
	AI.QE.5 Represent real-world problems using quadratic equations in one or two variables and solve such problems with technology. Interpret the solution(s) and determine whether they are reasonable.	High	Students will solve real-world problems that model quadratic functions using technology, and understand the meaning of the solution (s) within the context of the situation.
	AI.QE.6 Graph exponential and quadratic functions with and without technology. Identify and describe key features, such as zeros, lines of symmetry, and extreme values in real-world and other mathematical problems involving quadratic functions with and without technology; interpret the results in the real-world contexts.	High	Student will understand, interpret, and graph exponential and quadratic functions that represent real-world models, with and without technology. Students will interpret key function characteristics of the graphs in the context of the real-world model.
	AI.QE.7 Describe the relationships among a solution of a quadratic equation, a zero of the function, an x-intercept of the graph, and the factors of the expression. Explain that every quadratic has two complex solutions, which may or may not be real solutions.	Medium	Students will conceptually understand the difference between the zero of a function, solution to an equation, and the x-intercept of an expression using a graph, table or an equation, with OR without technology.

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Big Math Ideas

Geometry

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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May 2022

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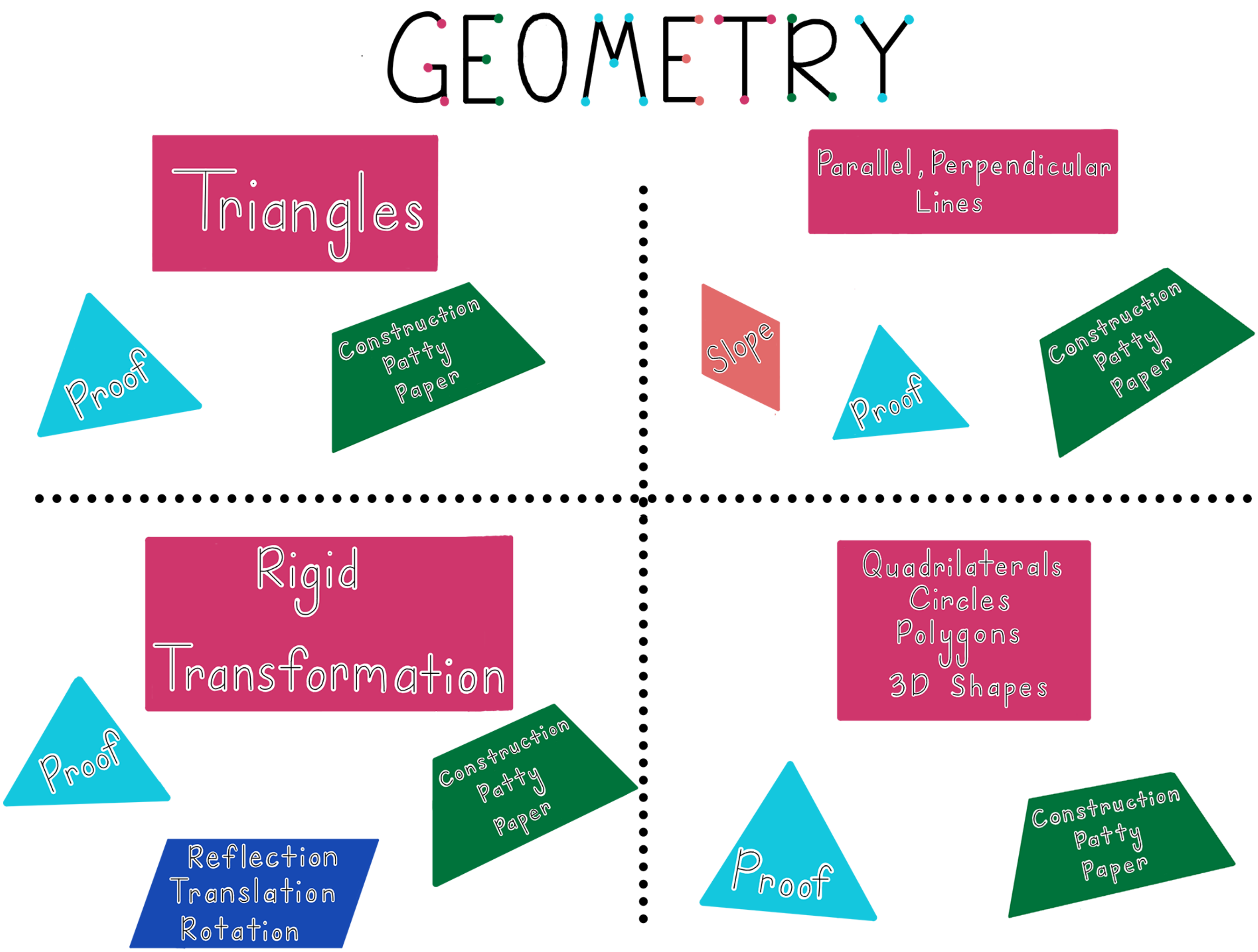
Geometry Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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Geometry Big Math Ideas - Narrative

The 2022 Geometry course emphasizes a conceptual understanding of 4 Big Math Ideas:

- Triangles
- Rigid Transformations
- Parallel/Perpendicular Lines
- Quadrilaterals/Circles/Polygons, 3D Shapes

These 4 Big Math Ideas are taught and learned using reasoning and proofs and informal constructions.

Rigid Transformations

After a brief introduction of the key definitions of geometry, such as point, plane, line, etc. and a few days spent on logic statements, such as the converse and contrapositive of a statement, students are ready for rigid transformations. Rigid transformations include: rotation, reflection, and translation;all of these are taught inside and outside a coordinate plane. Educators should introduce each transformation through the use of informal constructions using patty paper (Sample patty paper translation). These patty paper constructions allow students to conceptually visualize how each transformation actually transforms the shape. Students can begin to connect transformations and study real-world connections such as how programmers and coders create video games (CK-12 basic explanation). There are numerous real-world applications to rigid transformations. Also, informal reasoning and proof with rigid transformations are used to deepen student understanding. Do not include dilation, since it is not a rigid transformation, in this unit because this will be studied in the similar triangles unit with scale factor.

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Parallel/Perpendicular Lines

The second Big Math Idea in the Geometry course is parallel and perpendicular lines. Again, educators should introduce the definitions, theorems, and postulates associated with these geometric concepts through patty paper, paper folding, or informal constructions. Students will be able to visualize the "why" of the geometric postulates and theorems, rather than simply a list of things' to memorize. Learning should incorporate all types of proofs and reasoning into this unit. Students should experience paragraph proofs, two-column proofs, flowchart proofs, and any combination of logical, sequential reasoning leading from a "given" to a ""prove." Students should not be assessed on any one type of proof; rather, they should be able to choose the method to deliver their "proof" pathway. Incorporating real-world models into these tasks often allow students to see the meaning of the mathematics.

Proofs

Students will use proofs in each of the 4 Big Math Ideas in geometry. However, the teachers should want to encourage the use of a variety of "proof pathways" such as: paragraph proofs, two-column proofs, flowchart proofs, and any combination of logical, sequential reasoning leading from a "given" to a "prove." Why move away from a more standardized model of proof, like a two-column proof? The conceptual understanding behind a proof in geometry is the student's ability to demonstrate the use of geometric "truths" (definitions, postulates, theorems) in a logical way, given certain information, to prove something is true. If the end goal is truly understanding, then why does it matter what pathway or method students use to get there? For some students, a two-column proof may feel too rigid; for some students, a graphic organizer might better help them organize their thoughts. We want to encourage the logical progression of their thoughts, not hinder their thinking by boxing them into a particular method the teacher likes or the textbook uses. Educators should present the options to students and be open to any ideas they may develop on their own to demonstrate their understanding of geometric reasoning.

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Triangles

Students will study the concepts of congruence, similarity, proportionality, and inequality with triangles. Students should always use constructions (patty paper, paper folding/cutting, informal) to prove each postulate or theorem. It is critical that students conceptually understand why the "rule" exists and can explain, in their own words, the theorem or postulate as well as utilize it in an informal proof pathway. The triangle standards are not all the same priority level, so refer to the standards priority standards that appear later in this document to determine if the concept is a high or medium priority within the triangles standards. Real-world models should be utilized extensively throughout the study of triangles. Trigonometric ratios are not a high priority standard for geometry, and therefore this topic should not take up a considerable amount of time, as may be suggested by some textbooks. Law of Sines and Law of Cosines are not taught in geometry, yet they will be taught in precalculus for those students who choose that math course pathway.

Quadrilaterals/Circles/Polygons, 3D Shapes

As students begin their study of quadrilaterals, the Big Math Idea is related to the relationships, similarities and differences, and characteristics within the family of quadrilaterals. The study of the family of quadrilaterals should **never** focus on memorizing the definition and a set of "things" which are true about each quadrilateral. As with all of the other Big Math Ideas in geometry, real-world models, proofs, and informal constructions are the key to the conceptual understanding of these standards. Students shouldn't learn a postulate or theorem anywhere in geometry without a conceptual construction or proof to help them know and understand the "why." This activity (*Mrs. E Teaches Math*) is an excellent example of a paper folding/cutting in-class activity which enriches students' understanding of mathematics and the "why." This foldable is NOT something which supports conceptual understanding for students. It isn't wrong to use this type of graphic organizer, but all it accomplishes is surface learning of definitions and rules. It doesn't help students to own and internalize the relationships, similarities and differences, and real-world models using geometric figures such as quadrilaterals, circles, polygons, and 3D shapes.

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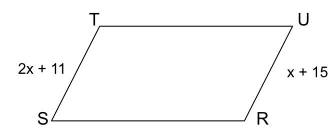
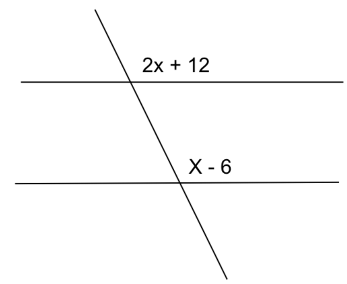
The majority of the standards in this domain are at a medium priority level and therefore do **not** need to be taught to mastery level. Students should be able to apply their knowledge to real-world situations which vary from the examples they have been shown and understand the relationships between mathematical concepts. They do not need to be able to apply their knowledge to new and unique situations as they do with mastery standards.

Algebra in Geometry

In more traditional textbooks and past standards documents (**not** in the 2020 Indiana Academic Standards), algebraic manipulations were a significant part of the geometry problems given to students. Consider the examples on the right.





In each of these examples, the algebraic manipulation is added to falsely layer complexity, but it detracts from the conceptual understanding of the geometric concepts. There are no references in the 2020 Indiana standards to adding algebraic equation work into geometric concepts. Instead, the focus should remain on real-world models and tasks which build conceptual understanding of the geometric concepts, such as in these examples:

- Locating Warehouses - *Illustrative Mathematics*
- Fun Size Cans - *Mathematics Assessments*
- Floor Pattern - *Mathematics Assessments*

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Geometry Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Logic and Proofs	**G.LP.1** Understand and describe the structure of and relationships within an axiomatic system (undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems). Understand the differences among supporting evidence, counterexamples, and actual proofs.	Medium	Taught in the context of other geometric content. Students can apply their understanding of the structures and use of the axiomatic system to the logic in geometric formal and informal proofs and reasoning. This standards is not assessed at any one point in time, but throughout the year.
	G.LP.2 Use precise definitions for angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, and plane. Use standard geometric notation.	Medium	Students understand the characteristics of each of the figures listed in the standard, can identify the figures, can construct (using patty paper, or informal construction tools, technology) and can utilize the figures in the context of other geometric problems and reasoning. This standard is NOT about memorizing a definition.
	G.LP.3 State, use, and examine the validity of the converse, inverse, and contrapositive of conditional ("if – then") and bi-conditional ("if and only if") statements.	Medium	Students are able to understand the appropriate use of the converse, inverse, and contrapositive of conditional statements.

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	G.LP.4 Understand that proof is the means used to demonstrate whether a statement is true or false mathematically. Develop geometric proofs, including those involving coordinate geometry, using two-column, paragraph, and flow chart formats.	Medium	Students will demonstrate a conceptual understanding of building a logical, progressive argument from a 'given' statement to a 'proven' statement using reasoning. Students may use a variety of methods including but not limited to: flow chart proofs, verbal proofs, patty paper proofs, two-column proofs, paragraph proofs, student developed proofs. These proofs should involve coordinate geometry.
Points, Lines, and Angles	**G.PL.1** Prove and apply theorems about lines and angles, including the following: a. Vertical angles are congruent. b. When a transversal crosses parallel lines, alternate interior angles are congruent, alternate exterior angles are congruent, and corresponding angles are congruent. c. When a transversal crosses parallel lines, same side interior angles are supplementary. d. Points on a perpendicular bisector of a line segment are exactly those equidistant from the endpoints of the segment.	High	Students will conceptually understand through the construction (patty paper or informal construction) of the proof of each of the theorems listed. Students will apply these theorems readily to logical geometric reasoning problems. Students should not memorize these theorems, this standard is about an **understanding of**, and a student's internalizing of, the concepts.
	G.PL.2 Explore the relationships of the slopes of parallel and perpendicular lines. Determine if a pair of lines are parallel, perpendicular, or neither by comparing the slopes in coordinate graphs and equations.	Medium	Students will **geometrically** explore the relationships between the slopes of parallel and perpendicular lines and apply their understanding to the comparison of graphs and equations of lines to determine the relationship between the lines.

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	G.PL.3 Use tools to explain and justify the process to construct congruent segments and angles, angle bisectors, perpendicular bisectors, altitudes, medians, and parallel and perpendicular lines.	High	Students will construct (using patty paper, informal paper folding, technology) the following (see standards for list). Students will conceptually **understand/justify** how the construction of each is connected to its characteristics.
	G.PL.4 Develop the distance formula using the Pythagorean Theorem. Find the lengths and midpoints of line segments in the two-dimensional coordinate system.	Medium	Students will develop the distance formula from the Pythagorean Theorem. Students will understand how lengths and midpoints of line segments can be found in a two-dimensional coordinate system.
	G.T.1 Prove and apply theorems about triangles, including the following: a. Measures of interior angles of a triangle sum to 180°. b. The Isosceles Triangle Theorem and its converse. c. The Pythagorean Theorem. d. The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length. e. A line parallel to one side of a triangle divides the other two proportionally, and its converse. f. The Angle Bisector Theorem.	High	Students will prove, through construction (patty paper, paper folding, technology) or geometric proof, each of the listed theorems. Students will conceptually understand each of the theorems and be able to readily apply them to geometric problems and informal proofs. This standard is NOT about memorization, it is about understanding and making connections between the constructions and the proofs needed to create these geometric truths.

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	G.T.2 Explore and explain how the criteria for triangle congruence (ASA, SAS, AAS, SSS, and HL) follow from the definition of congruence in terms of rigid motions.	High	Students will conceptually understand, through construction (patty paper, paper folding, technology) or geometric proof, each of the listed theorems as it relates to the concept of rigid motion. Students will apply these theorems to geometric problems and informal proofs, while demonstrating a conceptual understanding through justification.
	G.T.3 Use tools to explain and justify the process to construct congruent triangles.	Medium	Students will understand, through construction (patty paper, paper folding, technology) the process and meaning of congruent triangles including their characteristics and how their construction is connected to standard G.T.2.
	G.T.4 Use the definition of similarity in terms of similarity transformations, to determine if two given triangles are similar. Explore and develop the meaning of similarity for triangles.	High	Students will conceptually understand similarity through construction (patty paper, paper folding, technology) and how it is connected to scale factor from 7th grade. Students will understand the characteristics of similar triangles, specifically through geometric, **real-world problems and informal proof.**
Triangles	**G.T.5** Use congruent and similar triangles to solve real-world and mathematical problems involving sides, perimeters, and areas of triangles.	High	Students will understand how to apply their deep understanding of congruent and similar triangles to real-world models and make sense of the solutions in the context of the situation.

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	G.T.6 Prove and apply the inequality theorems, including the following: a. Triangle inequality. b. Inequality in one triangle. c. The hinge theorem and its converse.	Medium	Students will conceptually understand the listed theorems through construction (patty paper or informal construction) or informal proof. Students will apply these theorems readily to logical geometric reasoning problems. Students should not memorize these theorems, this standard is about an understanding, and a student's internalizing of the concepts.
	G.T.7 Explore the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle. Understand and use the geometric mean to solve for missing parts of triangles.	Medium	Students will understand, through construction (patty paper, paper folding, technology) or informal proof, the relationship that exists when the altitude is drawn to the hypotenuse of a right triangle. Students will apply their understanding to geometric **real-world problems** involving geometric mean.
	G.T.8 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Medium	Students will understand, through informal proof, how the trigonometric ratios are defined.
	G.T.9 Use trigonometric ratios (sine, cosine, tangent and their inverses) and the Pythagorean Theorem to solve real-world and mathematical problems involving right triangles.	Medium	Students will understand and apply trigonometric ratios and the Pythagorean Theorem to real-world problems and make meaning of the solutions in the context of the problem. This standards is NOT about memorizing the ratios.

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	G.T.10 Explore the relationship between the sides of special right triangles (30° - 60° and 45° - 45°) and use them to solve real-world and other mathematical problems.	Medium	Students will understand, through construction (patty paper, paper folding, technology) or informal proof, the relationship between sides in (30° - 60° and 45° - 45°) special right triangles. Students can use that understanding to solve **real-world problems** and make meaning of the solutions in the context of the problem.
	G.QP.1 Prove and apply theorems about parallelograms, including those involving angles, diagonals, and sides.	Medium	Students will conceptually understand, through construction (patty paper or informal construction) or informal proof, each of the theorems about parallelograms. Students will apply these theorems readily to logical geometric reasoning problems. Students should not memorize these theorems, this standard is about an understanding, and a student's internalizing of the concepts.
	G.QP.2 Prove that given quadrilaterals are parallelograms, rhombuses, rectangles, squares, kites, or trapezoids. Include coordinate proofs of quadrilaterals in the coordinate plane.	Medium	Students will apply their understanding of quadrilaterals in informal proofs related to special quadrilaterals including those in the coordinate plane.

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Quadrilaterals and Other Polygons	**G.QP.3** Develop and use formulas to find measures of interior and exterior angles of polygons.	Medium	Students will understand, through construction (patty paper, paper folding, technology) or informal proof, the formulas to find the measures of interior and exterior angles of polygons. Students will use their understanding to solve geometric **real-world problems** and make meaning of the solutions in the context of the problem.
	G.QP.4 Identify types of symmetry of polygons, including line, point, rotational, and self-congruences.	Low	Students will identify the types of symmetry in polygons. This is a cursory exploration standard.
	G.QP.5 Compute perimeters and areas of polygons in the coordinate plane to solve real-world and other mathematical problems.	Medium	Students will solve real-world problems, in the coordinate plane, involving perimeter and area of polygons. Students will make meaning of the solutions in the context of the situation.
	G.QP.6 Develop and use formulas for areas of regular polygons.	Medium	Students will understand, through construction (patty paper, paper folding, technology) or informal proof, the formula for area of regular polygons. Students will use their understanding to solve geometric **real-world problems** and make meaning of the solutions in the context of the problem.

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Circles	**G.CI.1** Define, identify and use relationships among the following: radius, diameter, arc, measure of an arc, chord, secant, tangent, congruent circles, and concentric circles.	Medium	Students will understand, through construction (patty paper, paper folding, technology) or informal proof, the relationships among the following: radius, diameter, arc, measure of an arc, chord, secant, tangent, congruent circles, and concentric circles. Students will utilize their understanding with geometric reasoning problems and real-world models.
	G.CI.2 Derive the fact that the length of the arc intercepted by an angle is proportional to the radius; derive the formula for the area of a sector.	Low	Students will understand, through construction or informal proof, the fact that the length of the arc intercepted by the angle is proportional to the radius. Students will also derive the formula for the area of a sector through informal proof.
	G.CI.3 Explore and use relationships among inscribed angles, radii, and chords, including the following: a. The relationship that exists between central, inscribed, and circumscribed angles. b. Inscribed angles on a diameter are right angles. c. The radius of a circle is perpendicular to a tangent where the radius intersects the circle.	Medium	Students will conceptually understand, through construction (patty paper or informal construction) or informal proof, each of the theorems listed. Students will apply these theorems readily to logical geometric reasoning problems. Students should not memorize these theorems, this standard is about an understanding, and a student's internalizing of the concepts.

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	G.CI.4 Solve real-world and other mathematical problems that involve finding measures of circumference, areas of circles and sectors, and arc lengths and related angles (central, inscribed, and intersections of secants and tangents).	Medium	Students will understand how to apply their understanding of circles to real-world models and make sense of the solutions in the context of the situation.
	G.CI.5 Use tools to explain and justify the process to construct a circle that passes through three given points not on a line, a tangent line to a circle through a point on the circle, and a tangent line from a point outside a given circle to the circle.	Low	Students understand through construction (patty paper, technology) and proof the process of constructing a circle that passes through three given points not on a line, a tangent line to a circle through a point on the circle, and a tangent line from a point outside a given circle to the circle. Students will be able to explain the process and construction to another student.
	G.CI.6 Use tools to construct the inscribed and circumscribed circles of a triangle. Prove properties of angles for a quadrilateral inscribed in a circle.	Medium	Students will understand how to construct (patty paper, paper folding, technology) the inscribed and circumscribed circles of a triangle. Students will understand how to use their construction process to prove the properties of angles for a quadrilateral inscribed in a circle and solve geometric problems using these properties.

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Transformations	**G.TR.1** Use geometric descriptions of rigid motions to transform figures and to predict and describe the results of translations, reflections and rotations on a given figure. Describe a motion or series of motions that will show two shapes are congruent.	High	Students will conceptually understand rigid transformations of a given figure, be able to perform transformations, and predict and describe the results of the rigid transformations. Students will apply a motion or a series of motions that will show two shapes congruent to **real-world situations** such as the development of video games.
	G.TR.2 Verify experimentally the properties of dilations given by a center and a scale factor. Understand the dilation of a line segment is longer or shorter in the ratio given by the scale factor.	High	Students will conceptually understand, through experimentally verifying, the properties of dilations including those listed in the standard. Students will apply their understanding to **real-world situations.**
Three-Dimensional Solids	**G.TS.1** Create a net for a given three-dimensional solid. Describe the three-dimensional solid that can be made from a given net (or pattern).	Medium	Students will understand how to create a net given a 3D solid. Students can describe the features and characteristics of the created 3D solid created from the net.
	G.TS.2 Explore and use symmetries of three-dimensional solids to solve problems.	Medium	Students will understand how to apply the use of symmetries of 3D solids to **real-world situations.**
	G.TS.3 Explore properties of congruent and similar solids, including prisms, regular pyramids, cylinders, cones, and spheres and use them to solve problems.	Medium	Students will solve real-world problems involving the properties of congruent and similar solids, including prisms, regular pyramids, cylinders, cones, and spheres. Students will make meaning of the solutions in the context of the situation.

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	G.TS.4 Solve real-world and other mathematical problems involving volume and surface area of prisms, cylinders, cones, spheres, and pyramids, including problems that involve composite solids and algebraic expressions.	Medium	Students will solve real-world problems involving volume and surface area of prisms, cylinders, cones, spheres, and pyramids, including problems that involve composite solids and algebraic expressions. Students will make meaning of the solutions in the context of the situation.
	G.TS.5 Apply geometric methods to create and solve design problems.	Medium	Students will understand how design problems can be created and solved using geometric methods.

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Big Math Ideas

Algebra 2

A mathematician's approach to the Indiana Academic Standards

Developed by Keep Indiana Learning

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May 2022

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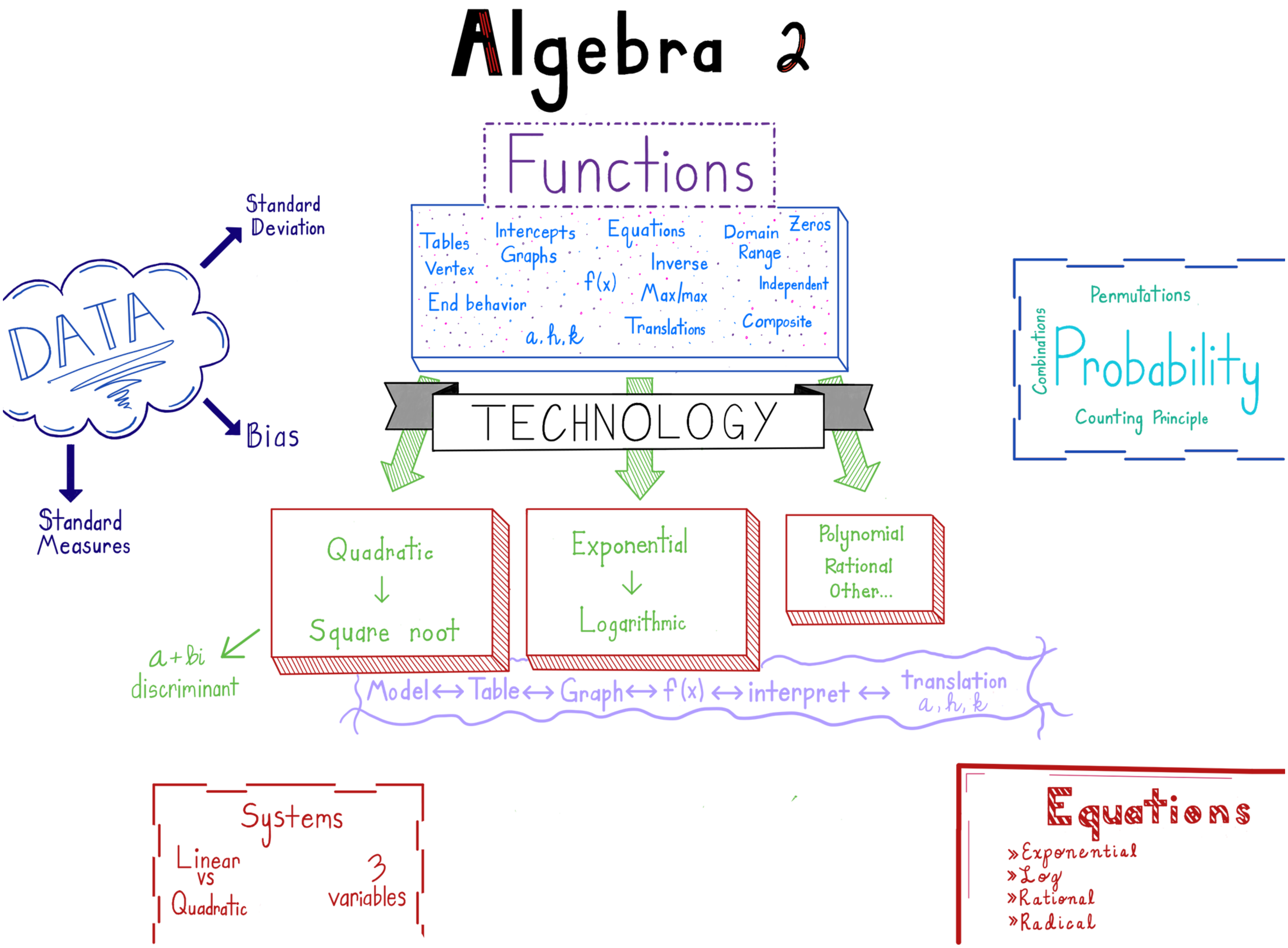
Algebra 2 Big Math Ideas - A Visual Representation

The visual representation of the Big Math Ideas highlights the connections, spotlighted concepts, and key learnings of the grade level in a image that aligns with the narrative and indicators of mastery.



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Algebra 2 Big Math Ideas - Narrative

Algebra 2 is a continuation of the study of functions that students began in Algebra 1. In Algebra 1, students learn that all functions have key characteristics in common such as: domain, range, max/min, intercepts, independent, dependent, rate of change, etc. and they also learn that functions have four basic representations, namely: graphs, tables, equations (function rule), and verbal model. Students learn to fluidly and flexibly move between these representations in Algebra 1 for linear, quadratic, and, to some extent, exponential functions. In Algebra 2, we will extend our function family to include the inverse of functions which will add logarithmic and square root functions. In addition, students will dig deeper into quadratic and exponential functions. In a cursory way, students will examine absolute value, polynomial, rational and piecewise functions. As we extend our function family in Algebra 2, it is critical that students continue to fluidly and flexibly move between graphs, tables, real -world models, and the function rule. They should be able to be given one of the 4 representations and determine the other 3 representations.

Algebra 2 Function Family

We begin the Algebra 2 course with a dive into the **inverse** of a linear function. By beginning the study of the inverse of a function with the linear function at the beginning of the year, students will get a brief review of the characteristics of a linear function and with a high priority standard (inverse of a function) that must be mastered by the end of the year. In addition, this is a great place to insert composite functions. The composite functions standard is a medium level standard, so not a Big Math Idea, but a good place to teach this concept. Once students understand the concept of a function's inverse, we can begin to study more functions and their inverses. In fact, moving forward, when students study a new function in the function family, we will study its inverse as well.

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Algebra 2 Functions

Type of Function	Characteristics	Representation of the functions	Priority in Algebra
Quadratic	Domain, range, intercepts, zeros, solutions, vertex, axis of symmetry, independent, dependent, increasing, decreasing, real-world model vs function rule, max/min, parent function, a,h,k translations	Fluidly and flexibly move between these 4 representations. Students can be given 1 representation and find the other 3. Table, graph, equation f(x), real-world model	High - Mastery required
Square Root	Domain, range, intercepts, increasing, decreasing, real world model vs function rule, max/min, parent function, a,h,k translations	Students should be able to work with all 4 representations using technology. Table, graph, equation f(x), real-world model	Exploratory
Exponential	Domain, range, intercepts, independent, dependent, increasing,	Table, graph, equation f(x), real-world model Fluidly and	High - Mastery required

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	decreasing, real world model vs function rule, max/min, parent function, a,h,k translations, growth, decay	flexibly move between these 4 representations. Students can be given 1 representation and find the other 3.	
Logarithmic	Domain, range, intercepts, independent, dependent, increasing, decreasing, real-world model vs function rule, max/min, parent function, a,h,k translations	Students should be able to work with all 4 representations using technology. Table, graph, equation f(x), real world model	Medium - Connecting real world models to the function rule to bring meaning to the students' understanding.
Polynomial, Piece-wise, and Absolute Value	Domain, range, intercepts, independent, dependent, increasing, decreasing, real- world model vs function rule, max/min, parent function, a,h,k translations	Students should be able to work with all 4 representations using technology. Table, graph, equation f(x), real -world model	Exploration. Students should recognize the parent function and graphs of each of these functions. Connections with real-world models are explored.

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The study of **quadratic functions** should come next in the scope and sequence of the course. In Algebra 1, students factored quadratic functions and expressions using the area model, and they solved quadratic equations by factoring, using the quadratic formula and graphing. However, in Algebra 2, we add completing the square to our toolbox of methods to solve a quadratic equation, and students will go much deeper in their understanding of a quadratic function and its behavior in the real world. For example, students will explore more deeply the vertex, axis of symmetry, intercepts, and domain/range of the situation vs. the function rule. The highest priority within the group of quadratic standards is the students' ability to understand the characteristics of a quadratic function and how it represents a real world model. Also an important priority is the students' ability to work with the parent function and translations (a, h, k) of the parent function. Additionally students will need to understand the value of each of the 3 forms of a quadratic equation (vertex form, standard form, intercept form). Of course students will need to be able to algebraically manipulate between the 3 forms of the quadratic equation as well. Similar to the approach in Algebra 1, the focus is the real-world model, **not** on factoring every type of quadratic equation. Students should not spend weeks reviewing factoring from Algebra 1. Instead, when starting the study on quadratic functions, students will jump in with real-world models and begin to explore the vertex, max/min, axis of symmetry, intercepts, domain and range for the situation in comparison to the function rule. Begin the study of quadratic functions with graphing using technology like Desmos. In Algebra 2, students should not waste time calculating a table by hand or graphing by hand very often. The point of the modeling is to see the behavior of the graph, not to practice graphing the points. Algebra 2 should be focused on the interpretation and conceptual understanding of the mathematical behaviors of the functions studied.

During the students' study of quadratic functions, they will solve the functions at $f(x) = 0$ and solve by completing the square, factoring, using the quadratic formula (including using the discriminant to determine the nature of the solutions, including complex numbers), and graphing. Keep in mind, the reason to solve is not so the student can master algebraic manipulations, rather it is to find the intercepts to interpret and make sense of the real-world model of the quadratic function.

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After a very thorough and mastery study of quadratic functions, students will explore the inverse, which is the square root function. This is not a function that must be mastered, therefore students will graph it, identify the parent function and translate it (a, h, k), and be able to identify the function rule modeled by a real world situation. They will need to solve the function by graphing and algebraically by squaring both sides and checking for extraneous roots. They must be able to identify what type of real-world situations fit a square root function and be able to understand and interpret the function characteristics of a square root function.

The next function, which is to be studied to mastery, in the scope and sequence of the course is the **exponential function.** We began the study of this function in Algebra 1 by introducing the parent function as both exponential growth and decay. Students explored real-world models of both growth and decay with graphing only. Our study of exponential functions will go much deeper in Algebra 2. Students will identify the graph of the parent function, translate it using a, h, k, and understand what real-world models fit the function. Students will be able to interpret situations using all of the function characteristics, including end behavior. Using applicable models, students will compare the real-world situation with the function rule, and solve exponential equations using technology.

The inverse of exponential functions is **logarithmic functions**, and that is the next function to study. Logarithmic functions are also a mastery standard; therefore students will identify the graph of the parent function, translate it (a, h, k), understand what real-world models fit the function, interpret situations using all of the function characteristics including end behavior, and compare the situation with the function rule.

After the work with the exponential and logarithmic functions, students will use the exponent laws to derive the logarithmic laws and evaluate expressions with exponents and logs. Students will also solve exponential and logarithmic equations with one variable.

The "other" functions students will study in Algebra 2 will include polynomial, piece-wise, and absolute value functions. Students do not learn these functions deeply. These are not functions to master; these are functions to **explore.** They will need to recognize the real-world models associated with each of these functions, characteristics of the functions, and be able to interpret those characteristics in the

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real-world model. Students will graph these functions with technology or sketch them if not using technology.

Data and Probability

The study of data in Algebra 2 is a **high priority** and should be placed at the beginning of semester 2. The data standards include a study of bias and inferences in data. Additionally, students do a deep dive into the statistical summary of data such as measures of center, spread, and outliers. Dig into some great real-world models with these standards. These are all high priority standards and should be studied in great depth.

The probability standards are equally important as the data standards for Algebra 2 students. The reason for this shift in priority for both data and probability is the shift in the number of college freshmen required to enroll in Calculus as a requirement of their major. In fact, only 10-16% of college freshmen take Calculus as part of a requirement for their degree. Most students will take courses like Quantitative Reasoning, Discrete Mathematics, and Finite Math where probability and data are often part of these courses; therefore, Algebra 2 students need to master the probability and data required standards. These standards include likelihood of a sample to make a prediction for a population (Three Act Math Task), Fundamental Counting Principle, and basic probability concepts. Dig deep into these high priority probability standards.

Systems of equations are not a Big Math Idea in Algebra 2, and, therefore, weeks should not be spent on this topic. Keep in mind, students will be solving a system of equations made up of a linear and quadratic equation, 2 linear inequalities, 2 linear equations, or 3 linear equations. All of these systems of equations should focus on real-world models. Graphing with and without technology, solving using substitution, and elimination are all options for students who should be given the opportunity to choose the method they want to use to demonstrate their understanding of systems of equations. By allowing students to choose their own method of solving a system of equations, the students are demonstrating their understanding of the

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mathematical concept which is the level of mastery needed; this is not a mastery level standard requiring students to show proficiency in all methods.

Minor concepts of Algebra 2

- Expanding fractional exponents into radical form and using the laws of exponents to rewrite expressions in these forms.
- Add, subtract, multiply and divide rational expressions. This is a minor standard; do not spend a considerable amount of time algebraically manipulating these expressions.
- Rewriting rational expressions in different forms such as this Illustrative Mathematics Task. Long division and synthetic division are no longer taught in Algebra 2.
- Solve absolute value linear equations and inequalities in one variable.
- Solve real world problems with rational equations; checking for extraneous solutions. Students should be able to interpret the solution(s) related to the real-world problem.

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Algebra 2 Big Math Ideas - Indicators of Mastery

Domain	Standard	Instructional Significance	Indicators of Mastery
Data Analysis, Statistics, and Probability	**AII.DSP.1** Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results.	High	Students will conceptually understand sampling such as random/non-random, controlling bias, characteristics of well designed sampling tools and making inferences from sampling results using **real-world models.**
	AII.DSP.2 Interpret and compare univariate data using measures of center (mean and median) and spread (range, interquartile range, standard deviation, and variance). Understand the effects of outliers on the statistical summary of the data.	High	Students will understand and interpret the meaning, in the context of **a real-world model,** the statistical summary of univariate data such as: mean, median, spread, and outliers. Limit problems that do not have a context. Technology should be used to calculate the statistical summary, the focus is on the interpretation in the context.
	AII.DSP.3 Use technology to find a linear, quadratic, or exponential function that models a relationship for a bivariate data set to make predictions; Interpret the correlation coefficient for linear models.	High	Students will identify, when given a real-world situation, the function (linear, quadratic, exponential) that best models the data set of bivariate data. Students will use their model to make predictions and interpret the correlation coefficient in the context of the situation for linear models. Students are using technology for all aspects of this standard.
	AII.DSP.4 Using the results of a simulation, decide if a specified model is consistent to those results. Construct a theoretical model and apply the law of large numbers to show the relationship between the two models.	Medium	Students will understand the law of large numbers through the use of comparing a theoretical model to a simulation of that model. The focus is on the understanding of the law of large numbers not the definition of the law.

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	AII.DSP.5 Understand dependent and independent events, and conditional probability; apply these concepts to calculate probabilities.	Medium	Students will conceptually understand probability, the role of independent and dependent events, and conditional probability. Students will demonstrate their understanding by calculating probabilities in real-world situations and explaining the results in context of the situation.
	AII.DSP.6 Understand the Fundamental Counting Principle, permutations, and combinations; apply these concepts to calculate probabilities.	Medium	Students will understand **how and why** the Fundamental Counting Principle, permutations, and combinations can be applied to calculate probabilities.
Arithmetic and Structure of Expressions	**AII.ASE.1** Explain how extending the properties of integer exponents to rational numbers allows for a notation for radicals in terms of rational exponents (e.g. 51/3) is defined to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.)	Low	Students will understanding how, using the laws of exponents, fractional exponents can be written in radical form.
	AII.ASE.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Medium	Students will use the laws of exponents to simplify expressions.
	AII.ASE.3 Rewrite algebraic rational expressions in equivalent forms (e.g., using properties of exponents and factoring techniques). Add, subtract, multiply, and divide algebraic rational expressions.	Medium	Students will add, subtract, multiply and divide **simple** algebraic rational expressions. Strategies such as expanded form, properties of exponents, factoring, and algebraic manipulation should be used to conduct the four operations.
	AII.ASE.4 Rewrite rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).	Low	Students will use algebraic manipulation to rewrite rational expressions. Long division and synthetic division should NOT be used.
	AII.F.1 Understand composition of functions and combine functions by composition.	Medium	Students will understand how functions are combined. Students will understand the results of the composition of functions related to the characteristics of functions such as domain and range.

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Functions	**AII.F.2** Define and find the inverse of a function. Verify functions are inverses algebraically and graphically.	High	Students will understand how to determine the inverse of a function and whether a function has an inverse.
	AII.F.3 Understand that if the graph of a function contains a point (a, b), then the graph of the inverse relation of the function contains the point (b, a); the inverse is a reflection over the line y = x.	High	Students will represent the table, graph, and equation f(x) of the inverse of a linear, quadratic, and exponential function based on their conceptual understanding of the inverse of a function.
	AII.F.4 Explore and describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative) with and without technology. Find the value of k given the graph of f(x) and the graph of f (x) + k, k f(x), f(kx), or f(x + k).	High	Students will understand the effects of the translations a, h, and k on the parent function of any graph.
Systems of Equations and Inequalities	**AII.SEI.1** Solve a system of equations consisting of a linear equation and a quadratic equation in two variables algebraically and graphically with and without technology.	Medium	Students will understand how to algebraically (substitution or elimination) and graphically (**with** and without technology) solve a linear/quadratic system of equations and the meaning of the solution in the context of a **real-world situation.**
	AII.SEI.2 Represent and solve real-world systems of linear equations and inequalities in two or three variables algebraically and using technology. Interpret the solution set and determine whether it is reasonable.	Medium	Students will understand how to solve and interpret the solution set to **simple,** real-world systems of linear equations and inequalities in two and three variables both algebraically and using technology.
	AII.SEI.3 Represent real-world problems using a system of linear equations in three variables. Understand that the algebraic steps to solve a two variable system can be extended to systems of equations in three variables.	Low	Students will create systems of linear equations in three variables that model real-world problems. Students will understand these systems can be solved using the same methods that solve systems with two variables (they do not need to solve them).

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Quadratic Equations and Functions	**AII.Q.1** Represent real-world problems that can be modeled with quadratic functions using tables, graphs, and equations; translate fluently among these representations. Solve such problems with and without technology. Interpret the solutions and determine whether they are reasonable.	High	Students will fluidly and flexibly represent quadratic functions with tables, graphs, equations f(x), and **real-world models.** Given one of the four representations of quadratic functions, students will develop the other three representations of the function with and without technology. These models include characteristics such as vertex, axis of symmetry, intercepts, rate of change, domain/range, max/min.
	AII.Q.2 Use completing the square to rewrite quadratic functions in vertex form and graph these functions with and without technology.	Medium	Students will use the area model to complete the square and rewrite a quadratic function in vertex form. Students will graph the quadratic functions in vertex form, with and without technology. Students will understand the benefit of a quadratic function written in vertex form.
	AII.Q.3 Understand that different forms of a quadratic equation can provide different information. Use and translate quadratic functions between standard, vertex, and intercept form to graph and identify key features, including intercepts, vertex, line of symmetry, end behavior, and domain and range.	Medium	Students understand the information available in of each of the three forms of a quadratic function equation. Students will understand the benefits of each form and when it is best to use each form for various situations. Students will be able to identify key information from the various forms of the function.
	AII.Q.4 Use the discriminant to determine the number and type of solutions of a quadratic equation. Find all solutions and write complex solutions in the form of $a \pm bi$ for real numbers a and b.	Medium	Students will understand why the discriminant identifies the number and type of solutions to a quadratic equation. Students will be able to use the discriminant to determine information about a given quadratic equation and explain the nature of the solutions.

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Exponential and Logarithmic Equations and Functions	**AII.EL.1** Graph exponential and logarithmic functions with and without technology. Identify and describe key features, such as intercepts, domain and range, asymptotes and end behavior. Know that the inverse of an exponential function is a logarithmic function.	High	Students will fluidly and flexibly represent exponential and logarithmic functions with tables, graphs, equations f(x), and real-world models. Given one of the four representations of each function, students will develop the other three representations of the function with and without technology. These models include characteristics such as asymptotes, end behavior, intercepts, translations, parent function, domain/range.
	AII.EL.2 Identify the percent rate of change in exponential functions. Classify them as representing exponential growth or decay.	High	Students will understand whether and why a real-world situation representing an exponential function represents growth or decay. Students will identify the percent rate of change of the function.
	AII.EL.3 Use the properties of exponents to rewrite expressions to describe transformations of exponential functions.	Medium	Students will use the laws of exponents to simplify exponential expressions to then describe transformations to the function.
	AII.EL.4 Use the properties of exponents to derive the properties of logarithms. Evaluate exponential and logarithmic expressions.	Medium	Students will understand how the properties of exponents can be used to derive the properties of logarithms. Students will understand how the properties can be applied to evaluate exponential and logarithmic expressions.
	AII.EL.5 Solve exponential and logarithmic equations in one variable.	Medium	Students will understand how the properties of exponents and logarithms can be utilized so solve simple equations.
	AII.EL.6 Represent real-world problems using exponential and logarithmic functions and solve such problems with technology. Interpret the solutions and determine whether they are reasonable.	High	Students will understand, represent and solve real-world situations that represent exponential and logarithmic functions. Students will interpret the solutions in the context of the situation.
	AII.PR.1 Solve real-world and other mathematical problems involving polynomial equations with and without technology. Interpret the solutions and determine whether the solutions are reasonable.	Medium	Students will understand, represent and solve real-world situations that represent polynomial equations, with and without technology. Students will interpret the solutions in the context of the situation.

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Polynomial, Rational, and Other Equations and Functions	**AII.PR.2** Graph mathematical functions including: a. polynomial functions; b. rational functions; c. square root functions; d. absolute value functions; and, e. piecewise-defined functions with technology. Identify and describe features, such as intercepts, domain and range, end behavior, and lines of symmetry.	Medium	Students will understand how to identify and graph the listed parent functions, **with technology**, identify and describe the key features related to that function.
	AII.PR.3 Solve real-world and other mathematical problems involving radical and rational equations. Give examples showing how extraneous solutions may arise.	Medium	Students will solve real-world problems involving radical and rational equations and demonstrate their understanding of extraneous solutions in the context of the situations.
	AII.PR.4 Solve absolute value linear equations and inequalities in one variable.	Low	Students will solve **simple** absolute value linear equations and inequalities in one variable.

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Reach out to the Authors

We are here for you! We can support you with the standards, curriculum mapping, instructional guidance, and curriculum alignment work. Reach out today!

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Dr. Laurie Ferry-Sales is the Director of Keep Indiana Learning at CIESC. She has served as an Indiana High School and Middle School Principal, Assistant Principal and Executive Director of Mathematics Teaching and Learning for several national organizations serving urban, high-need schools. Laurie was the K-12/Math Specialist for the Indiana Department of Education. She has served on national and state level professional organizations related to mathematics curriculum and instruction, next generation assessments and professional development. Laurie began her education career as a secondary mathematics teacher and later as a K-12 mathematics curriculum specialist.

Laurie brings over 32 years of educational experience to Keep Indiana Learning. She has presented at many state, national and international conferences, as well as provided professional development to over 20,000 educators during her career. Laurie's areas of expertise include mathematics curriculum and instruction, teacher effectiveness (RISE certified trainer), instructional coaching support, administrator training around teacher feedback and teacher retention, ILearn and SAT assessments, and building professional development models for schools.

Laurie received her BS degree in Secondary Mathematics Education from Concordia University in Chicago, IL. She earned her MS in Educational Leadership and her Principal Certification from Purdue University. Dr. Ferry-Sales dissertation was related to the impact math professional development can have on teachers' practice. Connect with Laurie on Twitter @LaurieAFerry or via email at lsales@ciesc.org.

Courtney Flessner

Courtney Flessner spent 17 years working in public, private, and public charter schools in Indiana, New York City, and Wisconsin. She has been a classroom teacher in grades 1-6 and 7th and 8th grade mathematics. Courtney has also served as an instructional coach, administrator and taught elementary math methods at IUPUI for seven years.

Courtney has a Master of Arts in Educational Policy and Politics and a Master of Education in Educational Policy from Teachers College at Columbia University. Currently, she is working to complete her PhD from Indiana University in Educational Leadership and Policy and Mathematics Education.

Courtney prioritizes working with schools to help them better understand what instructional leaders need to do to support the creation of environments where elementary teachers of mathematics feel supported in implementing and sustaining ambitious and equitable mathematics teaching practices in their classrooms. She is determined to change the narrative of admitting that one is not a "math person" by inspiring teachers to teach ambitiously - and maybe even have a little fun while doing so.

Courtney is the proud wife and colleague of Dr. Ryan Flessner, a professor in the College of Education at Butler University and is the mama of Abel and Adelyn who attend Washington Township Schools.

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Jeff Harker

Jeff Harker is a 32 year veteran secondary mathematics educator of public schools in Indiana. During his tenure he cultivated his role as teacher leader in the classroom and out of the classroom as a cross country coach for 30 years. He was also a literacy coach for 5 years during his career. He has presented at the school, district, state and national levels on topics such as struggling learners, questioning to engage learners, grading for learning, reading strategies, differentiation, and student self-efficacy. He has always enjoyed his role as teacher, both for students and adults.

Jeff obtained his Bachelor's degree in Mathematics from Ball State University. (Chirp chirp!) Then went on to receive a Master's degree in Secondary Education from Indiana University. After spending a year teaching in North Carolina, he returned to his roots in Northern Indiana at Warsaw Community Schools and a short stint teaching for Ivy Tech, finally ending up in the Indianapolis area in Lawrence Township where he spent the last 25 years. After retiring from teaching he found his way back to his passion for professional development and helping teachers at CIESC.

As a Professional Learning Specialist with Keep Indiana Learning and CIESC, Jeff is focused on helping teachers hone their craft and helping their students become independent, successful mathematicians.

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Jessica Miller

Jessica Miller has been in the field of education since 2008. She started her career as an elementary classroom teacher before spending three years as her elementary school's STEM Teacher/Coach. Most recently, Jessica served as an Instructional Coach where she supported teachers through coaching cycles and provided professional development at the building and district levels.

Jessica earned her undergraduate degree in Elementary/Middle Childhood Education from Butler University and received her educator license as an Elementary/Intermediate Generalist and in Elementary/Intermediate Mild Intervention in 2008. She earned an Elementary/Middle School Math Specialist Masters Degree from Ball State in 2019 and received an Elementary Math Specialist addition to her license in 2021.

Jessica values creating partnerships with teachers and schools with a focus on enhancing instructional practices in the areas of math, classroom best practices, and technology integration. She works collaboratively with teachers and teams through professional learning experiences that prioritize ambitious, student-centered instruction.

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